Current detectors, noise and control.

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World map of ground-based interferometric detectors



Outline

- I) Basic elements
- II) External mechanical disturbance: suspended Optics
- III) Feednack control basics
- IV) Locking
- V) Noise hunting and performance
- VI) Enjoying digital control in practice
- VII) Noise hunting and performance
- VIII) Focus on monolithic suspension







First generation of interferometric GW detectors





Basic elements

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I) Free-test-mass differential detector in practice

Wide-band detection

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(=> more detectable sources !!)
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possible through interferometry.



Michelson ITF, response to GW

$$ds^{2} = -c^{2}dt^{2} + (1 + h_{+}(t))dx^{2} + (1 + h_{+}(t))dy^{2} + dz^{2} \Longrightarrow \frac{dx}{dt} = \pm \frac{c}{\sqrt{1 + h_{+}(t)}}$$

$$\int_{t_1}^t \frac{dt'}{\sqrt{1+h(t')}} = \frac{1}{c} \left(\int_0^{l_1} dx - \int_{l_1}^0 dx \right) = \frac{2l_1}{c}$$

along x

$$\int_{t_1}^t \left(1 - \frac{1}{2}h(t')\right) dt' = (t - t_1) - \frac{1}{2} \int_{t - \frac{2l_1}{c}}^t h(t') dt' \approx \frac{2l_1}{c}$$

in the same way
$$\varphi_x(t) = \omega t_1 \approx \omega \cdot \left(t - \frac{2l_1}{c} - \frac{1}{2} \int_{t - \frac{2l_1}{c}}^t h(t') dt'\right)$$
$$\varphi_y(t) = \omega t_2 \approx \omega \cdot \left(t - \frac{2l_2}{c} + \frac{1}{2} \int_{t - \frac{2l_2}{c}}^t h(t') dt'\right)$$

if
$$l_1 = l_2 = l$$
 $\Delta \varphi(t) \approx \omega \int_{t - \frac{2l}{c}}^{t} h(t') dt' = \Delta \varphi_{GW}(t)$

Antenna Pattern for free-mass detectors

$$\frac{\Delta L}{L} = \frac{\Delta L_1 + \Delta L_2}{L} = \frac{1 + \cos^2 \theta}{2} \cos(2\varphi) h^+ + \cos\theta \sin(2\varphi) h^x$$



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λ [deg]

Beam pattern function averaged over polarization

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(ecliptic coords)

Network of gravitational wave detectors as a global instrument

Coherent analysis with several detectors: crucial to detect the signal and to reconstruct the 5 parameters of the wave





incidence direction (ϕ, θ)



For example, coherent combination advanced LIGO-H/LIGO-L/Virgo (SNR=8, eff.90%), enlarges the BNS detection distance at which BNS from 150-170 Mpc of the
single antennas → 270 Mpc

Michelson ITF, basic formalism (I)



Michelson ITF, basic formalism (II)

Given a GW source at distance r the effect due to optimal coupling of polarization "+" is:

$$\begin{split} l_1 & \rightarrow l_1 \left(1 + \frac{1}{2} h_+ \right) \\ l_2 & \rightarrow l_2 \left(1 - \frac{1}{2} h_+ \right) \end{split}$$

$$\begin{split} P_{out} &= \frac{P_{in}}{2} \left(1 + C\cos(2k \cdot \delta l) + Ck(l_1 + l_2) \cdot h_+ \sin(2k \cdot \delta l) \right) \\ &= P_{Mich.} + \delta P_{GW} \qquad \delta P_{GW} \propto Amplitude_{GW} \end{split}$$

ITF **power** signal is sensitive to the **amplitude** of gravitational waves (and not to their **power**, as in e.m. wave detection).

ITF GW signal fades as 1/r (and not as $1/r^2$ as in e.m. telescopes): the number detectable sources increases as Δ Sensitivity³

Sensitivity intrinsically limited by the noise source affecting the power at the detection port: **shot noise**. Dark-fringe (P_{out}^{min}) ensures the best SNR

A very important parameter: the contrast



Shot noise, the intrinsic readout noise (Vinet's lesson)



In terms of linear spectral density :

Actual cases tuning, shot noise*

In terms of power spectral density the SNR is

$$\frac{S_{GW}(f)}{S_{shot}(f)} = \frac{P}{\hbar\omega}(r_1^2 + r_2^2) \cdot C \cdot F(2k\delta l, r_1, r_2) \cdot S_x(f)$$

Sharp function of δl around the dark fringe



Read-out: Simple Michelson low noise detection concept



At the detection port we have: $\Psi_{out}^m(t) = \Psi_0 + \Psi_+ e^{i\Omega t} + \Psi_- e^{-i\Omega t}$

$$\begin{split} \Psi_{0} &= -J_{0} \left(e^{i\delta l \omega_{0}/c} + e^{-i\delta l \omega_{0}/c} \right) & - \text{global phase neglected.} \\ \Psi_{\pm} &= \pm J_{1} \left(e^{i\delta l (\omega_{0} \pm \Omega)/c} + e^{-i\delta l (\omega_{0} \pm \Omega)/c} \right) e^{\pm i(l_{1}+l_{1})\Omega/c} & - \delta l \equiv \Delta l + \delta l_{gw} \\ \text{arm-length asymmetry} \\ + \\ \text{GW} \end{split}$$

Detected power:
$$|\Psi_D|^2 \approx |\Psi_0|^2 + |\Psi_-|^2 + |\Psi_+|^2 + |\Psi_+\Psi_0^* - \Psi_0\Psi_-^*|e^{i\Omega t} + |\Psi_-\Psi_0^* - \Psi_0\Psi_+^*|e^{-i\Omega t}$$

$$= DC + A \cdot \sin \frac{\Omega}{c} \Delta l \sin 2 \frac{\omega_0}{c} \delta l_{gw} \cdot (A_1 \sin \Omega t + A_2 \cos \Omega t)$$
A,A₁ e A₂ constants
A small ITF arm-asymmetry is necessary
in order to detect the strain signal (in-fase at Ω)

II) Designing ITF configuration in real cases

By exploiting edge-technology solutions it is possible in principle to reach 10^{-21} - 10^{-23} Hz^{-1/2} strain sensitivity over a quite large bandwidth.

Expected rate of coalescences: 3/yr within 40 ÷ 200 Mpc [Grishchuk et al. Astro-ph/0008481]

Coalescence event rate at ~ 20 Mpc [Kalogera et al. ApJ. 601, L 179, 2004] -0.3/yr for NS/NS



Estimated rate of SNe: several /yr in the Virgo cluster (20 Mpc).



II) Designing ITF configuration in real cases

By exploiting edge-technology solutions it is possible in principle to reach 10^{-21} - 10^{-23} Hz^{-1/2} strain sensitivity over a quite large bandwidth.



Virgo Final Design 1998

An ambitious project

• Ground-based detector of extragalactic GW whose bandwidth starts from low frequency (few Hz - few kHz).





Further reduction of intrinsic read-out noise sources

I) Given dark-fringe operation, higher SNR: $\cos(2k\delta l) = -1$

II) Shot noise target (L=optical path, P=power)

$$Sh_{shot}^{1/2} \cong 3 \cdot 10^{-23} \, / \sqrt{Hz}$$

III) Technical solutions:

=> Fabry-Perot cavities on the ITF arms:

$$S\phi_{shot}^{1/2} = \sqrt{\frac{2\hbar\nu}{P}} \Rightarrow Sh_{shot}^{1/2} = \frac{\lambda}{4\pi L}\sqrt{\frac{2\hbar\omega}{\eta \cdot P}}$$

ms: $L = L_{FP} \frac{2F}{\pi}; F = 50, L_{FP} = 3$ km

$$\frac{Recycling factor}{P = G_{Rec}} P_{in}; G_{Rec} = 50, P_{in} = 20W$$

=> ITF reflected light recycled:

=> Basics: see next slides..

Laser beam/cavity basics (I)

 $\nabla^2 U(x,y,z) + k^2(x,y,z) = 0$ Coherent monocromatic wave emitted by the source

Along z, Hermite-Gauss complete set:

$$U(x,y,z) = \frac{A_{mn}}{w(z)} H_m(\sqrt{2}\frac{x}{w(z)}) H_n(\sqrt{2}\frac{y}{w(z)}) + \exp\left[-\frac{x^2 + y^2}{w^2(z)} - i\frac{k(x^2 + y^2)}{2R(z)} - i(kz - \phi_{mn})\right]$$

$$w^{2}(z) = w_{0}^{2} \left[1 + \left(\frac{\lambda z}{\pi w_{0}^{2}} \right)^{2} \right]$$

beam divergence θ_{∞}
$$R(z) = z \left[1 + \left(\frac{\pi w_{0}^{2}}{\lambda z} \right)^{2} \right]$$

$$\phi = (m + n + 1) \arctan(\theta_{\infty} \frac{z}{w_0})$$

Laser beam/cavity basics (II)

In a mode-matched cavity, if the cavity is stable and R(z) and w(z) are the same after an arbitrary number of reflections,



FP basics (I) (plane waves)

 $\psi_{in} = K e^{i\chi}$ $\psi_1 = unknown$ $\psi_2 = e^{-ikl}\psi_1$ $\psi_3 = ir_2\psi_2$ $\psi_{A} = e^{-ikl}\psi_{A}$ steady state: $\psi_1 = t_1 \psi_{in} + i r_1 \psi_4$

 $\frac{\psi_{in}}{\psi_r} \frac{\psi_1}{\psi_4} \frac{\psi_2}{\psi_3}$ l = lMirror 1 Mirror 2

 $\psi_1 = \frac{\iota_1}{1 + r_1 r_2 e^{-2ikl}} \psi_{in}$ $\psi_{4} = \frac{ir_{2}t_{1}}{1 + r_{1}r_{2}\rho^{-2ikl}}\psi_{in}$ **Reflected wave:** $\psi_r = ir_1\psi_{in} + t_1\psi_4 = i\frac{r_1 + r_2e^{-2ikl}}{1 + r_1r_2e^{-2ikl}}\psi_{in}$ Transmitted wave: $\psi_t = t_2 \psi_2 = \frac{t_1 t_2 e^{-ikl}}{1 + r_1 r_2 e^{-2ikl}} \psi_{in}$ stored wave

FP basics (II)

$$\begin{aligned} \left|\psi_{r}\right|^{2} &= \left|A_{r}\right|^{2} \\ \left|\psi_{t}\right|^{2} &= \left|A_{t}\right|^{2} \end{aligned} \qquad A_{r} = \sqrt{\frac{r_{1}^{2}r_{2}^{2} + 2r_{1}r_{2}\cos 2kl}{1 + r_{1}^{2}r_{2}^{2} + 2r_{1}r_{2}\cos 2kl}} \\ \left|\psi_{t}\right|^{2} &= \left|A_{t}\right|^{2} \end{aligned} \qquad A_{t} = \sqrt{\frac{1}{1 + r_{1}^{2}r_{2}^{2} + 2r_{1}r_{2}\cos 2kl}} t_{1}t_{2} \end{aligned}$$

$$L = (2m+1)\frac{\lambda}{4} \implies A_r = \min = \frac{r_2 - r_1}{1 - r_1 r_2}; A_t = MAX = \frac{t_1 t_2}{1 - r_1 r_2}$$
if:

$$L = m\lambda \qquad \longrightarrow \qquad A_r = MAX = \frac{r_2 + r_1}{1 + r_1 r_2}; A_t = \min = \frac{t_1 t_2}{1 + r_1 r_2}$$

FP basics (III)

The stored wave amplitude ratio is $\frac{\psi_1}{\psi_{in}} = \frac{t_1}{1 + r_1 r_2 e^{-2ikl}}$



FP basics (IV)

Given a resonance condition for λ_0 $L = (2m+1)\frac{\lambda_0}{4} \Rightarrow 2k_0 l = (2m+1)\pi$

=> Frequency separation between peaks (FSR)





Absorption peak (reflected)



Absorption peak (transmitted)

FP basics (V) (optical length...towards Michelson intererometer)

$$\psi_{r} = i \frac{r_{1} + r_{2}e^{-2ikl}}{1 + r_{1}r_{2}e^{-2ikl}}\psi_{in} \qquad \varphi_{r}(l) = \arctan\left(\frac{r_{1}(1 + r_{2}^{2}) + r_{1}(1 + r_{2}^{2})\cos(2kl)}{r_{2}(1 - r_{1}^{2})\sin(2kl)}\right)$$
$$\varphi_{r}(l_{res} + \delta l) = \varphi_{r}(l_{res}) + \frac{d\varphi_{r}}{dl} \left| \delta l = \varphi_{r}(l_{res}) - 2k \left(\frac{2F}{\pi}\right) \delta l$$

The response of phase of FP reflection to length variations is enhanced by its Finesse

Finite light speed to be taken into account

$$\omega_{c} = 2\pi \frac{FWHM}{2} = \frac{\pi c}{2Fl} = \frac{1}{\tau_{s}}$$
 cavity storage t
Frequency cut-off in the response to GW

FP basics (VI) (optical length...towards Michelson intererometer)

Ι

$$|\psi_{out}|^2 = \frac{|\psi_{in}|^2}{2} (1 + C\cos(2kl))$$

$$|\psi_{out}|^2 = \frac{|\psi_{in}|^2}{2} (1 + C\cos(2kl + \varphi_{r-FP2} - \varphi_{r-FP1}))$$



For a given optimal polarization:

$$\left|\varphi_{r-FP2} - \varphi_{r-FP1}\right| = 2k \frac{\frac{2F}{\pi}L_0}{\sqrt{1 + \left(\frac{\omega_{GW}}{\omega_c}\right)^2}} \cdot h$$

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Optical length of cavities depends on the frequency!



FP-Michelson ITF-power recycling II (power built-up and losses)

Recycled power $|\psi_0|^2$ implicitly depends on

⇒Recycling cavity length l_r ⇒Michelson recombination phase ϕ_{Mic}



Power rec. mirror reflectivity choice depends on ITF and Recycling cavity losses:

$$|\psi_0|^2 = \frac{1}{p_{\text{Re}c}^2 + p_{ITF}^2} |\psi_{in}|^2$$

UBS

Intrinsic optical noise sources: power and frequency noise sources

- To get rid of the power fluctuation effects:
 - Interferometer (cavity, fringe and recycling) locked with the Pound-Drever-Hall technique (heterodyne at about 10 Mhz);
 - Signal extracted through partial sidebands transmission (Schnupp technique);

To have sidebands partially transmitted an asymmetry of the Michelson is necessary (about 0.8 m in Virgo);

The arm asymmetry produces laser frequency noise;

- The frequency fluctuations requirements are about 10⁻⁴ Hz/sqrt(Hz) at 1 Hz. A complex frequency stabilization is necessary:
 - Input mode cleaner filtering fluctuations
 - Rigid frequency cavity reference

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- Laser frequency locking on the interferometer common mode **OPORTORING STORY**

v9

Standard layout used in ground-based detectors (i.e. framework upon Advanced detectors are being based)

I) The optical path where the GW-induced phase shift is accumulated can be enhanced by means of Fabry-Perot resonant cavities.

II) Dark fringe detection to reduce read-out noise.

Effective path =120 km FP: higher finesse phase of refl. light (pi) 0.5 simple mirror Ω -0.5 FP: lower finesse -1 LASER -1.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4 05 round-trip phase (pi) Beam splitter The response of the ITF is much steeper => control system needed EMVESF270710 Photo detector

III) Recycling of ITF reflection.
FP lock concept

Widely used in radiophysics (R.Pound 1946), and later on adopted in GW detector optics (R. Drever), nowadays it is the standard method for cavity lock and laser stabilization.





At resonance length incident light is reflected with $-\pi/2$ phase at carrier frequency and with $\pi/2$ at sidebands.

$$E_r(t) = -E_0 \cdot \cos(\omega_0 t) - mE_0 \cos(\Omega t) \cdot \sin(\omega_0 t)$$

A small change in cavity length :

$$E_r(t) = -E_0 \cdot \cos(\omega_0 t + \phi_d) - mE_0 \cos(\Omega t) \cdot \sin(\omega_0 t + \phi_d)$$
$$\left| E_r(t) \right|^2 \propto m \cdot \sin \phi_d \cos \Omega t$$

EMVESF2 In-phase demodulated signal at Ω is driven by length variations



Summarizing shot noise

Simple Michelson

$$Sh_{shot}^{1/2}(t) = \frac{1}{2\pi} \frac{1}{2L} \sqrt{\frac{\hbar c\lambda}{\eta P_{in}}}$$

Michelson +Fabry-Perot

$$Sh_{shot}^{1/2}(t) = \frac{1}{2\pi} \frac{1}{2L_{opt}} \sqrt{\frac{\hbar c\lambda}{\eta P_{in}}} \sqrt{1 + \frac{\omega^2}{\omega_c^2}}$$

Michelson +Fabry-Perot +Power recycling

$$Sh_{shot}^{1/2}(t) = \frac{1}{8FL} \sqrt{\frac{hc\lambda}{\eta G_{\text{Re}c} P_{in}}} \sqrt{1 + \frac{\omega^2}{\omega_c^2}}$$

Г

back-action noise

$$Sx_{RP} = \frac{1}{m \cdot 2\pi v_0^2} \frac{SP_{shot}^{1/2}}{c}$$

displacement power spectrum due to radiation pressure on suspended mirrors

$$Sh_{overall}^{1/2} = \frac{1}{L_{FP}} \sqrt{\frac{c_{shot}}{F^2 P} + \frac{c_{RP}}{(mv_0^2)^2}} F^2 P$$

m = suspended mirror mass; $c_{shot} = c_{shot} (const_{shot}); c_{RP} = c_{RP} (const_{RP})$



Intrinsic test mass noise source: thermal

• The test masses are a part of a large ground-based digitally controlled mechanical system meant to keep them in quasi-inertial state.

• The spatial coordinate x of such a mechanical chain is probed by the ITF at the mirror.

• Each test-mass is not point-like and then at thermal equilibrium *x* fluctuates due to thermal impulse fluctuation within its body.

$$SF_{xx} = 4kT \cdot \Re\{Z_{xx}\}$$

$$SX_{xx}^{2} = \frac{4kT}{\omega^{2}} \cdot \Re\left\{Z^{-1}_{xx}\right\}$$

Z is the mechanical impedance of the system; it has to be known (theory or measurement) in order to be inverted. Close to a mechanical resonance \tilde{x}_{TN} is a Lorentian. Phenomenological models are often used (...) for internal friction noise predictions.

$$Sh_{TN}^{1/2} = \frac{\sqrt{SX_{TN}}}{L} = \frac{1}{L} \int \frac{4kT}{\omega_o^2} \frac{1}{(\omega_o^2 - \omega_o^2)^2 - (\phi\omega_o^2)^2}}{\frac{1}{Mass Temperature, dissipation}}$$



Virgo-like approach in TAMA and GEO



LIGO



Passive (to reduce noise in sensitive freq. band)



Active (to improve lock acquisition/maintenance)



Suspended mirror speed, orders of magnitude



Suspended mirror position fluctuation

The mechanics of SA suspension is designed to reach 10⁻¹⁸ m/Hz^{1/2} at 10 Hz (thermal noise)



• The SA filters off the seismic noise above 4 Hz

- Below 4 Hz the mirror moves at the SA resonances few tens of μm
- ITF locking requires resonance damping

TOP: Sophisticated control system for the suspension chain

BOTTOM: Efficient and noiseless payload control

SA

meter



Susp-gallery





Local Controls: Inertial Damping

- Inertial sensors (accelerometers):
 - DC-100 Hz bandwidth
 - Equivalent displacement sensitivity: 10⁻¹¹ m/sqrt(Hz)
- Displacement sensors LVDT-like:
 - Used for DC-0.1 Hz control
 - Sensitivity: 10⁻⁸ m/sqrt(Hz)
 - Linear range: ± 2 cm
- Coil magnet actuators:
 - Linear range: ± 2 cm
 - 0.5 N for 1 cm displacement
- Loop unity gain frequency:
 5 Hz
- Sampling rate: - 10 kHz



Virgo "standard-super-attenuator" to allow LF sensitivity Soft isolator concept: very efficient passive attenuation 1. active controls for suspension mode 2. Inertial damping damping to the SA Inverted quasi-inertia magnets digital control pendulum actuators $f_0 = 40 \text{ mHz}$ Ground-based sensors magnets Mechanical filters 7.5 m CCD: 6 d.o.f. $A = 10^{-14}$ PSD: θ_x, θ_y UHV @10Hz. laser PSD: z, θ_x, θ_y Steering stage (marionette) test mass $\delta \theta_{\rm x} = \delta \theta_{\rm y} < 0.1 \ \mu rad RMS, \ \delta z = 0.5 \ \mu m RMS$ **Drift** $(1 h) \sim 1 \mu rad$

Range: 5x10⁴-5x10⁻² μrad,10⁴-0.1 μm

Suspension digital control (9 stations): ~all controls are operated using suspended actuators

I) Local controls apply corrections to mirror position using local sensors: swinging interference

II) Local controls receive error signals from global sensors.ITF Locked, resonant light



Orientate suspended optics with respect to local ground reference



Optical diagonalization of optical levers allow to reduce the coupling among mirror d.o.f. to <1% => Large dynamics: 50 mrad < 80 nrad thanks to hybrid sensor system

Two states of operation of mirror suspension control

I) Incoherent (Local Control) : the mirrors are controlled by means of independent ground-based sensors and quasi-inertial actuators.

- \rightarrow relative position of mirrors is correlated by the ground
- **II**) **Coherent** (Global Control): the operation point has to be locked by using ITF light. Angular control performed coherenty with respect to the light into the ITF by means of beam wavefront sensing (i.e. detection of $\text{TEM}_{01,10}$ modes due to misalignment.

 \rightarrow relative position of mirrors is correlated by light

Functions of suspension stages and control configuration: the Virgo case (TAMA, LCGT similar): 1) single point suspension, 2) DOF separation, 3) inertial damping, 4) hierarchical control \rightarrow small forces close to the test mass.



Basic requirements: sensing and actuation diagonalization + hiearchical control Divide et impera!



III) Feed-back control basics

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In order to deduce the effect of the feedback switch block schematization can be used.

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*feed-forward is also used in several subsystems ITFs, but is not mentioned here





$$out(s) = ref(s)\frac{P(s)}{H(s)}\frac{T(s)}{1+T(s)} + dist(s)G_2(s)\frac{1}{1+T(s)}$$

 $T(s) = G_c(s)G_1(s)H(s)$ Loop gain

$$\frac{out(s)}{dist(s)} = \frac{G_2(s)}{1 + T(s)}$$
In absence of input driving the disturbance transfer function is reduced by increasing the loop gain $T(s)$

$$in(s)=0$$

$$\frac{out(s)}{ref(s)} = \frac{P(s)}{H(s)} \frac{T(s)}{1+T(s)} \approx \frac{P(s)}{H(s)}$$
The capability to follow the reference with high-gain loops makes the system stuck by $H(s)$

Considering a noise on
$$H(s)$$

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$$ut(s) = \frac{ref(s)P(s) - m(s)}{H(s)} \frac{T(s)}{1 + T(s)} + dist(s)G_2(s)\frac{1}{1 + T(s)}$$

Useful definitions and relations



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credits A. Gennai

Feedback Specifications

- The most common approach to feedback design is to work directly on **open-loop return ratio G(s) K(s)**. Usual specification are the following:
 - Abs(G(s) K(s)) > L >> 1, for $0 \le f \le f_1$
 - $Abs(G(s) K(s)) < \epsilon \ll 1$, for $f > f_h$
 - Gain margin > m , Phase margin > ϕ
- Above specifications are usually converted in the following one:
 - "Use high loop gain over some frequency range then decrease the gain as rapidly as possible" keeping into account that:
 - An Open-loop stable the roll-off rate in the unity gain region must not exeed 40 dB/dec; for a reasonable phase margin it should be smaller ...
 - Right half-plane zeros impose an upper limit to unity gain frequency
 - Right half-plane poles impose a lower limit to unity gain frequency

Stability

• Stability of a feedback system is determined by the roots of the closed loop *characteristic equation* (i.e. the poles of closed loop transfer function)

1+G(s) K(s)=0

- Nyquist Stability Theorem
 - The closed loop system is stable if and only if the graph of G(jω) K(jω) in the complex plane, for -∞ < ω < ∞, encircles the point -1+j0 as many times anticlockwise as G(s) K(s) has right half-plane poles



• Analog to Digital (ADC) and Digital to Analog Converters (DAC) are the interface between the 'Analog' and the 'Digital' world.

Time-continuous vs time-discrete controllers

- Why digital controllers?
 - Programmability and Flexibility
 - Stability and Repeatability
 - Performances
- Why analogue controllers?
 - Bandwidth
 - Dynamics

The Z-Transform

• The Z-transform of a discrete-time function *f(kT)* is designated by F(z) and is defined as follows:

$$F(z) = \sum_{k=-\infty}^{\infty} f(kT) z^{-kT}$$

• Bilinear Transformation

Analog Prewarp
$$S = \frac{2}{T} \frac{(1-z^{-1})}{(1+z^{-1})}$$
 • • Oigital

Timing And Time Delay

- Timing has a key role in any digital control system.
- The sampling period shall be constant since poles and zeros position are a function of sampling period. Varying the period means varying poles and zeros locations
- ADC and DAC must be synchronized not to introduce phase noise.

Sensing & Driving (implemented in DSP controllers)



M(s) => 'Sensing' : projection of sensors output along given (preferred) directions D(s) => 'Driving' : projection of correction signal along actuators directions C(s) => Compensator

D and M implemented in Virgo are 'static' matrixes (constant) such that C(s) is diagonal, hence: Multiple Input / Multiple Output (MIMO) converted to a set of Single Input /Single Output (SISO)



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The operation point





- Keep the FP cavities in resonanceMaximize the phase response
- Keep the PR cavity in resonance
 Minimize the shot noise
- Keep the output on the "dark fringe"
 - Reduce the dependence on power fluctuations

Keep the armlength constant within $10^{-12} m$!
Alignment configuration for Virgo (VSR1)

Note: Automatic alignment, based on wavefront sensing and DC asymmetry is a aommon practice in all interferometers (a specific chapter should be dedicated to the topic)



Basic interference setup: two conditions



→ Common Arm length (CARM)

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Basic interference setup: two conditions



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concept:

ULE rigid REF. CAVITY→ LASER@LF 2 stage Fstab CARM \rightarrow LASER(a) HF



E.g.: requirements for Virgo G. Vajente 2010

D.O.F.	Requirements [m]	Accuracy [m]		
		VSR1	May 2008	
DARM	3 · 10 ⁻¹¹	$3 \cdot 10^{-12}$	$1 \cdot 10^{-15}$	
PRCL	$2 \cdot 10^{-10}$	$3 \cdot 10^{-11}$	$1\cdot 10^{-12}$	
MICH	$5 \cdot 10^{-10}$	$8 \cdot 10^{-11}$	$3 \cdot 10^{-11}$	
CARM		$4\cdot 10^{-8}$	$2\cdot 10^{-8}$	

Sensing

- How do we actually measure lengths?
 - Laser light is phase-modulated: we have a 'carrier' frequency and two sidebands
 - Carrier is resonant in the two long cavities
 - Sidebands are resonant in the recycling cavity
 - The demodulated photodiode output is a function of phase shift (i.e. a length variation) in the resonant field.
- Motion along the 4 degrees of freedom is detected using three photodiodes (for B5 we use both phase and quadrature components)

(PRCL)		(0	$g_{\rm PRCL}$	0	0) ($B5_Quad$
MICH		$g_{\scriptscriptstyle MICH}$	g_X	0	0		B2_Phase
CARM		0	0	0	$g_{\scriptscriptstyle CARM}$		B1_Phase
		0	0	$g_{\scriptscriptstyle DARM}$	0) (B5_Phase

Driving

- MICH, PRCL and DARM are corrected applying forces on 4 mirrors
- CARM is corrected using a sort of trick
 - CARM error signal is used to change the laser frequency: in this way laser frequency swings following the common mode motion of the two long cavities
 - At low frequency laser frequency is 'locked' to a reference cavity length thus preventing CARM to drift away.

$$\begin{pmatrix} F_{BS} \\ F_{PR} \\ F_{NE} \\ F_{NE} \\ F_{WE} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 0 & 0 \\ -2 & -1 & 0 \\ 0.1 & 0 & 1 \\ -0.1 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} F_{MICH} \\ F_{PRCL} \\ F_{DARM} \end{pmatrix}$$

Basic controller for Virgo

- A digital controller is used for MICH PRCL and DARM.
- An analog controller is used for CARM
- Dynamic compensators are quite complex. We have between 10 and 20 poles for each degree of freedom.
- Unity gain frequencies are in the 10 50 Hz range for MICH PRCL and DARM and in the 100-200 kHz range for CARM



Locking strategies strongly depend on suspension systems: LIGO and Virgo (strategies will be significantly different for Adv Detectors)





Control noise → smallest forces close to test masses

- Required locking accuracy:
- $dL \sim 10^{-12} \, m$
- Tidal strain over 3 km: $dL \sim 10^{-4}$ m
- Wide dynamic range to be covered without injecting actuation noise.
- Hierarchical Control
- 3 actuation points

<u>A note about hierarchical control:</u>
Acting on Marionette from the last suspension stage, Superattenuator modes are excited (blue)
Acting on Mirror from Reference Mass only, one longitudinal mode is excited (green)



Actuation on the test masses: full hierarchical control

19h58

19h58



Switch to low noise coil drivers

Virgo Locking in 3 blocks

Basic concept: controlling the reflectivity of non-recycled interferometerby adding a **DC offset** on BS position. W PR Ν NPUT NE BEAM BS $r=r_{FP}\cos(k\Delta l_{\perp})$ СМ PR NPUT BEAM $r \approx 1$ $r \approx 0$ DARK BRIGHT FR NGE FRINGE

credits L. Barsotti

COMPOUND MIRROR

WE

Concetto base: un piccolo ricircolo di potenza altera poco le risposte usate per controllare l'ITF



- PRCL resonant but mirror slitghtly **DC tilted** (3f used)
- MICH half free fringe
- CARM

Dalla frangia grigia alla frangia scura





DF signal (dark fringe) to control DARM



Overall locking is quite longer with respect to LIGO: LF suspensions imply several preparatory intermediate steps (11), however, the Virgo duty cycle is satisfactory.



Thermal Compensation Scheme (concept): its operation is integrated with locking and compensates cavity power losses due to mirror deformation



Thermal effects in the Virgo mirrors

(M. Punturo)

• The 3km Virgo arms are a long Fabry Perot cavity:



- Hence, actually, each arm is a double FP cavity:
 - Etalon effect $r_M = -r_{AR} + \frac{t_{AR}^2 r_{HR} e^{-2ikL_M}}{1 + r_{AR} r_{HR} e^{-2ikL_M}}$ $r_{FP} = -r_M + \frac{t_M^2 r_{end} e^{-2ikL_A}}{1 + r_M r_{end} e^{-2ikL_A}}$

Etalon Effect

(M. Punturo)

- Hence, the Finesse of the cavity and all the fundamental parameters of the ITF are affected by the input mirror optical thickness variation
- But, why the mirror optical thickness fluctuates?

• Temperature!!
$$\phi = k \cdot x = \frac{2\pi}{\lambda} n \cdot x \rightarrow \Delta \phi = \frac{d\phi}{dT} \Delta T = \frac{2\pi}{\lambda} \left[\frac{dn}{dT} x + n \frac{dx}{dT} \right] \Delta T$$

$$\Delta \phi = \frac{2\pi}{\lambda} x \left[\frac{dn}{dT} + (n-1) \cdot \alpha \right] \Delta T$$

- Hence, knowing the mirror temperature it is possible to predict some of the ITF performances
- OK, but how to measure the mirror temperature?

Resonant mode technique

(M. Punturo)

- Obviously the resonant frequencies of a body depend on the temperature of the body
- For a Virgo mirror we evaluated this dependence with a ANSYS based FEM





IV) Enjoying digital control in practice (with suspensions and interferometer together) (11)

An example



The example of inverted pendulum control



The case of Virgo is general for detectors aimed to low-frequency sensitivity

top-stage position RMS of actual inertial damping is affected by
non-perfect sensing acceleration sensing
Seismic reinjection through the control.



briefly... (sea activity versus quiet): combining sensors in order to reduce the impact *(O)* of ground disturbance injected by suspension position sensor is important !



Position/Acceleration blending: suspension quasi-inertial damping



A: two different crossover enabled in various environmental conditoins



Position/Acceleration prefiltering for top-stage control



fx @ 50 mHz, 'trade-off' with improved features

two main issues

The problem of improving the system through independent optimization of each suspension:

wind-tilt noise (through accelerometers, f < 70 mHz)

μseism sea disturbance (through position control, 0.15-0.6 Hz)

Trade-off: f_x=50 mHz

Position/Acceleration prefiltering for top-stage control



wind-earthqukes, f <70mHz: "aggressive" attenuation of accelerometer tilt noise.

two main issues

The problem of improving the system through independent optimization of each suspension:

wind-tilt noise (through accelerometers, f < 70 mHz)

μseism sea disturbance (through position control, 0.15-0.6 Hz)

Trade-off: f_x=50 mHz

Position/Acceleration prefiltering for top-stage control



two main issues

The problem of improving the system through independent optimization of each suspension:

wind-tilt noise (through accelerometers, f < 70 mHz)

μseism sea disturbance (through position control, 0.15-0.6 Hz)

Trade-off: f_x=50 mHz

µseism, 150-600 mHz: "aggressive", slightly worsened against tilt noise.

µseism disturbance attenuated downstream (→the concept of Global Inverted Pendulum Control)



The lock force applied to the marionette corrects the residual payload motion, whose rms above 100 mHz is \sim 1 order of magnitude smaller than the ground motion.

A top-stage control position reference with smaller seismic noise allows to increase position/accel crossover without risks HOW TO DO IT ?

µseism-free and Global Inverted Pendulum control

(0)

µseism is incoherent
along the arm baseline
=>µSeism reduced at
END suspension top-stages
by using position referred to
INPUT mirrors (GIPC);

µseism is coherent Among close suspensions. µSeism-Free control signals can be reconstructed with respect to INPUT mirrors (µSF)



INPUT TOWERS USED AS REFERENCE ALLOWS TO USE microSEISMIC-FREE SIGNALS TO CONTROL ALL THE OTHER TOP-STAGES

VII) Noise hunting and performance

(4)







V+ commissioning is interlaced with coincidence operation of enhanced-LIGO

VSR2

Science Mode duty cycle > 80%



Commissioning activities compliant with LIGO

Locked duty cycle > 85%



Interferometer working mode particion

Sensitivity improvements during commissioning and scientific runs



Background activity: four years spent to prepare preliminary Advanced Virgo design



Virgo+, first step towards Vadv: main improvements (many others underlaying)



Thermal noise: new paylaod Virgo+ <u>with</u> monolithic suspension and high-finesse-high-Q mirrors (current commissioning)


if $\Phi(\omega) = \Phi_{\circ}$



All materials

Dissipation measurements



High **Q** values

=> energy around the resonance (a benefit for any kind of detector of small forces)





$$k(\omega) = k_e (1 + i\phi_s(\omega)) \quad ; \quad k_e = \frac{\sqrt{TYJ}}{L^2} \quad ; \quad J = \frac{\pi}{4} r_w^4$$

$$\omega_p^2 = \left(\frac{k_e}{m} + \frac{g}{L}\right) = \omega_g^2 + \omega_e^2$$

$$-\omega^2 \theta + \frac{g}{L} \theta + \frac{k_e}{m} \left(1 + i \phi_s(\omega) \right) \theta = \frac{F_{ext}}{mL} \longrightarrow \text{FDT...}$$

Dissipation occurs around the wire bending point

thermoelastic losses (suspension wire bending)



 $\omega >> 2\pi/\tau$ \Rightarrow adiabatic compression/expansion $\omega << 2\pi/\tau$ \Rightarrow isothermal compression/expansion $\omega \approx 2\pi/\tau$ \Rightarrow max

Objects with very asymmetric aspect ratios as fibers and membranes are critical

surface losses (wires, fibers, ... clamping, mirrors)

$$\phi_{layer} = \frac{W_{surf}}{W_{tot}} \phi_{surf}$$

Inhomogeneity passing from the bulk to the surface causes elastic energy redistribution process.

$$\frac{W_{surf}}{W_{tot}} = \frac{\mu_w h_{sb} S_w}{V_w}$$

 μ_{w} form factor (for a wire) h_{sb} thickness of the layer $S_{w} = 2\pi r_{w} L_{w}$ external surface considered $V_{w} = \pi r_{w}^{2} L_{w}$ volume





Internal modes (mirrors a "sane" Brownian motion)

$$SX_{TN} = \frac{4k_bT}{\omega} \sum_{i} \frac{\omega_i^2 \phi_i}{M_i} \frac{1}{\left(\left(\omega_i^2 - \omega^2\right)^2 + \left(\omega_i^2 \phi_i\right)^2\right)}$$

Naive model, ok around resonance peaks

$$M_i; \quad \frac{1}{2}M_i\omega_i^2 X^2 = E_i$$

equivalent mass of mode *i*

 $\phi_i = \phi_{str} + \phi_{coat}$

coating losses important



Simulation NI/WI: 5584.9 Hz Measured NI: (5585.7 ± 0.5) Hz WI: (5583.5 ± 0.5) Hz NE: (5543.2 ± 0.5) Hz WE: (5545.6 ± 0.5) Hz

EMVESF270710

NE/WE: 5546.1 Hz



A relevant issue for AdV detectors !

Virgo, mechanical TN



HW experiments !





Transparence effects















Active-suspensions in the trolley !
Data-logger to monitor humidity and temperature
H-V accelerometers









