# Accelerometer Noise Budget

Matteo Passuello

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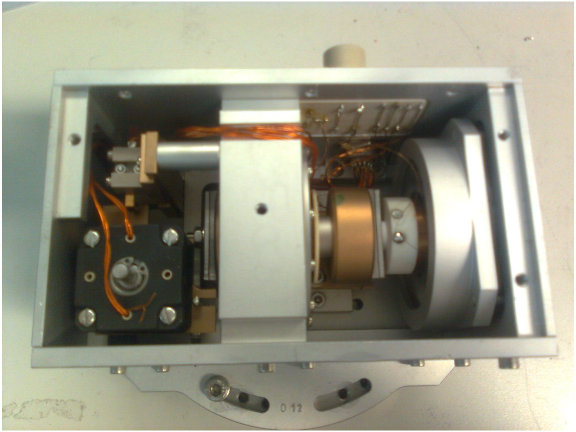
# Table of contents I

# Table of contents II

# Introduction: accelerometer



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# Object

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- Provide specifications for the new accelerometer generation to improve the efficiency also with the new electronic digital design

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# Dynamic Equations and TFs

# Dynamic accelerometer equation (1)

Dynamic accelerometer equations: translation (superimposition effects)

$$\ddot{x} + \gamma\left(\dot{x} - \dot{x}_0\right) + k\left(x - \dot{x}_0\right) = F, \frac{k}{m} = \omega_0^2, \frac{\gamma}{k} = \frac{1}{Q\omega_0}$$

$$[F = 0]\, x_0 \rightarrow x = \frac{\frac{s}{Q\omega_0} + 1}{\frac{s^2}{\omega_0^2} + \frac{s}{Q\omega_0} + 1}$$

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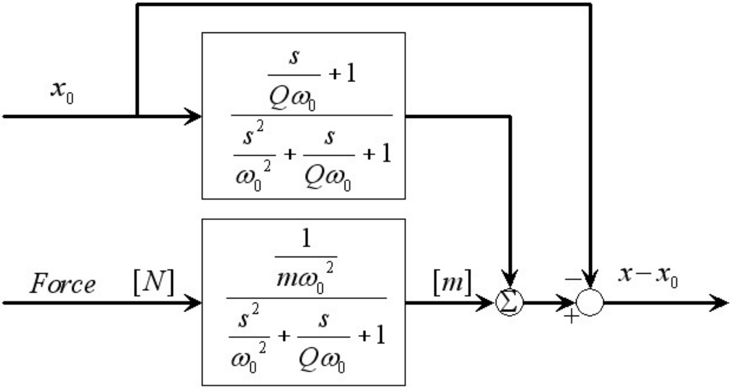

# Dynamic accelerometer equation (2)

Dynamic accelerometer equations: tilt

$$m\ddot{x} + \gamma\left(x - x_0\right) + \left(k - \frac{mg}{L}\right)\left(x - x_0\right) = -kL\alpha + F_{est}$$

with:

$$\frac{k}{m} - \frac{g}{L} = \omega^2 \rightarrow k = m\left(\omega_0^2 + \frac{g}{L}\right)$$

thus:

$$x_{\alpha \rightarrow x - x_0} = L\frac{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} - \frac{g}{L\omega_0^2}}{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1}$$

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# Sensing and Controller TFs

- *Sensing*: Butterworth filter with high frequency double pole(@2.3KHz) (DC value obtained from calibration)
- *Controller*: PID with integrator and double zero (@1Hz) in order to compensate the mechanical resonance
  Gain adjusted in order to have the unit gain loop @200Hz (closed loop)

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Noises evaluation from the electronic control circuit model (sized as the real one). Only considered actually the high frequency values.

- Lvdt noise after amplifier and mixer(sensing block): $300nV/\sqrt{Hz}$

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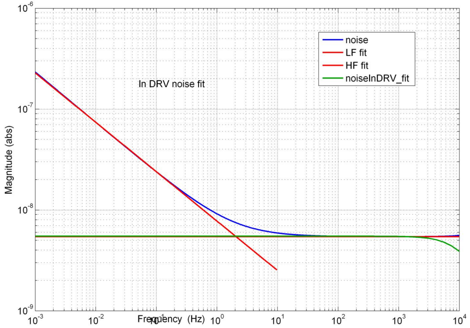
- Amplification noise on the output stage: $3.26nV/\sqrt{Hz}$



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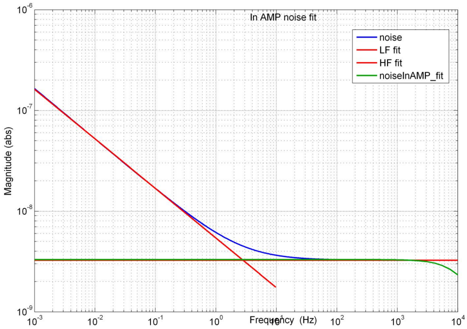

# Model's completion

- Evaluation of the magnet-feedback coil characteristic (block *DRV*): from the measure it's obtained the conversion factor $N/V$ to apply the correction signal from the *PID* controller to the actuator (force)
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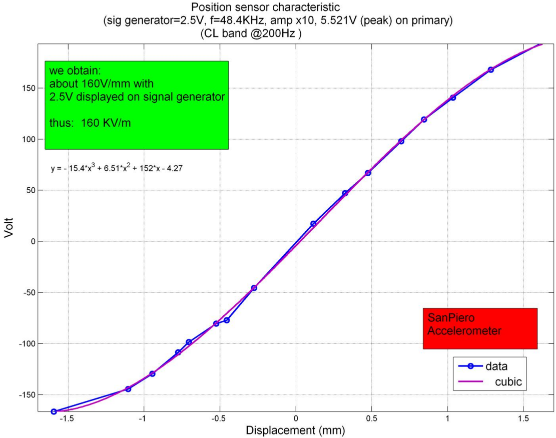
| Accelerometer | SanPiero | Virgo |
|---|---|---|
| $\omega_0$ | $1.375Hz$ | $3.375Hz$ |
| Q | $\approx 10$ | $\approx 25$ |
| $R$ fb coil | $\approx 193.11\Omega$ | $\approx 568\Omega$ |
| fb magnet weight | $572g$ | $572g$ |
| Out Amp | $\approx 9.2V/V$ | $\approx 9.2V/V$ |
| Mass estimate | $\approx 326g$ | $\approx 439g$ |

Position sensor characteristic

(Signal generator with $2.5V$, $48.4KHz$, amp$x10$, $5.512V$ peak on primary coil)

| SanPiero | Virgo |
|---|---|
| $\approx 160KV/m$ | $\approx 116KV/m$ |

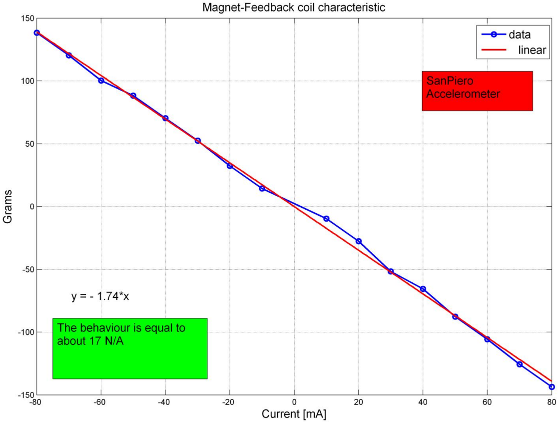
# Position sensor characteristic:



Magnet-Feedback coil characteristic:

| SanPiero | Virgo |
|---|---|
| $\approx 17N/A$ | $\approx 30N/A$ |

# Magnet-Feedback coil characteristic:



# Accelerometer Model

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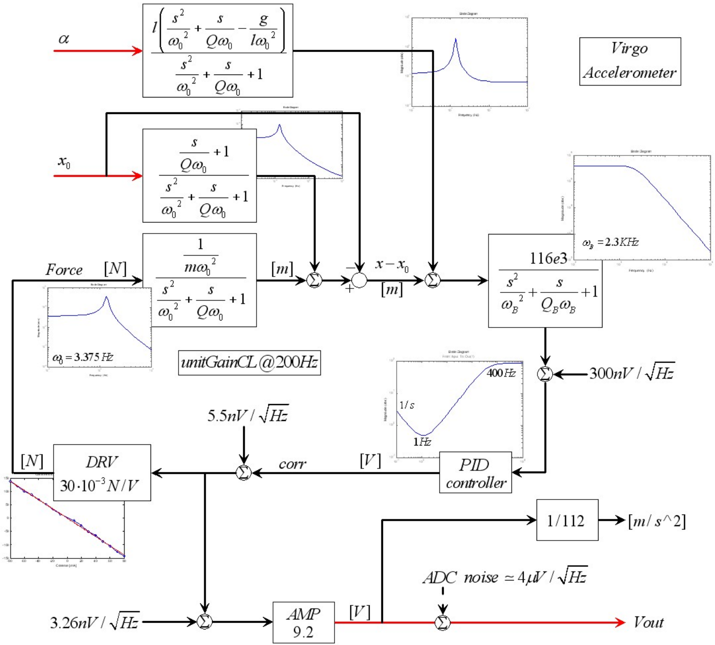
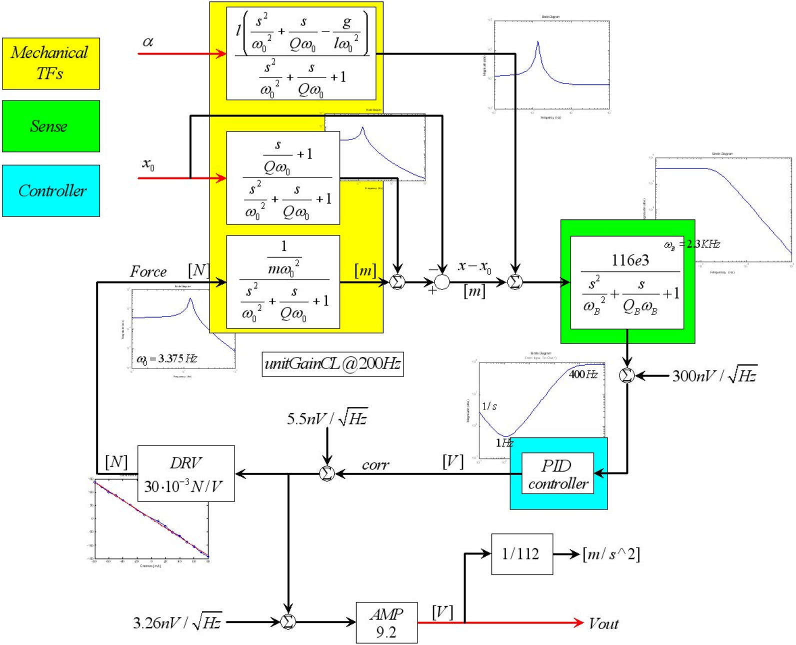
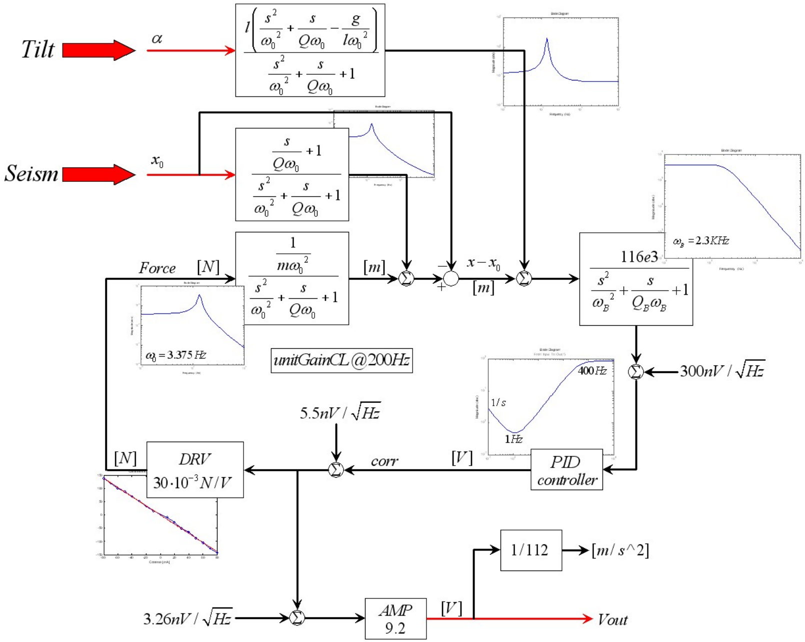


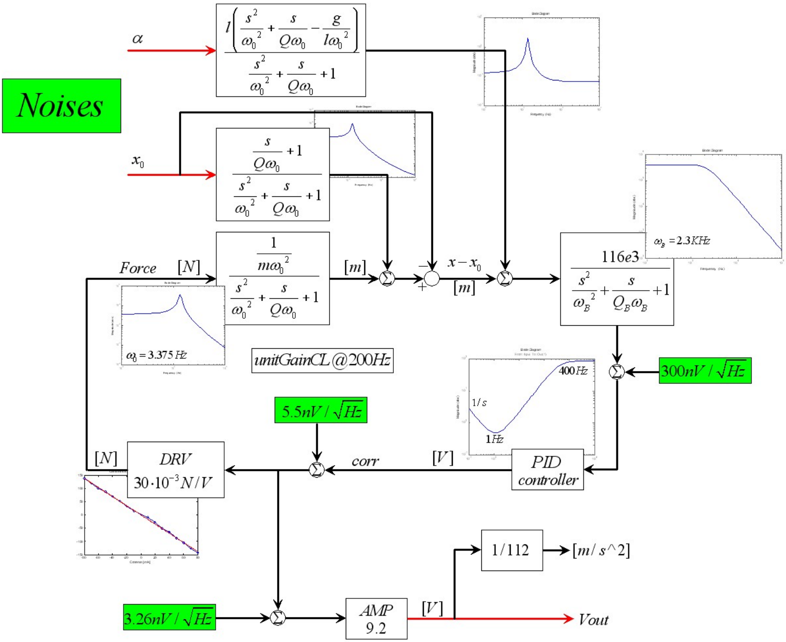
# Transfer functions



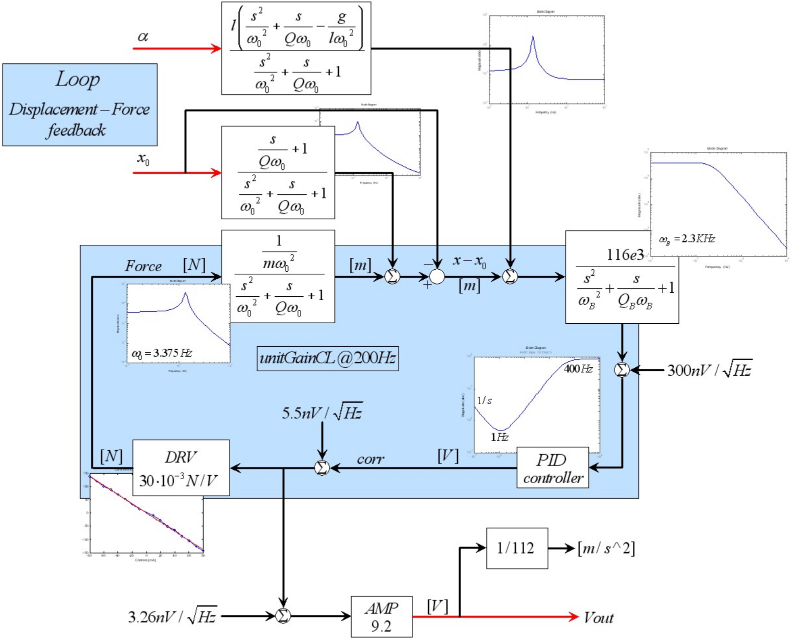
# Exogenous Inputs



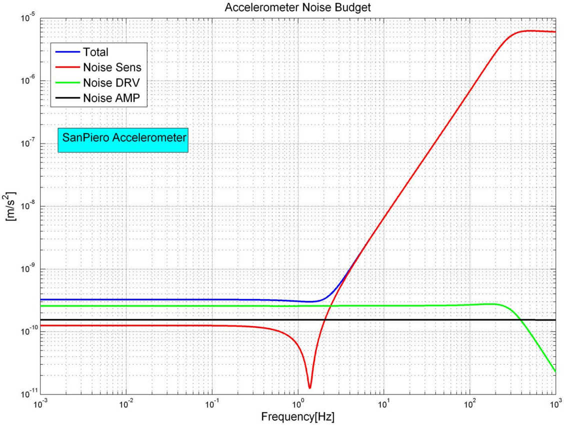
# Noise Sources



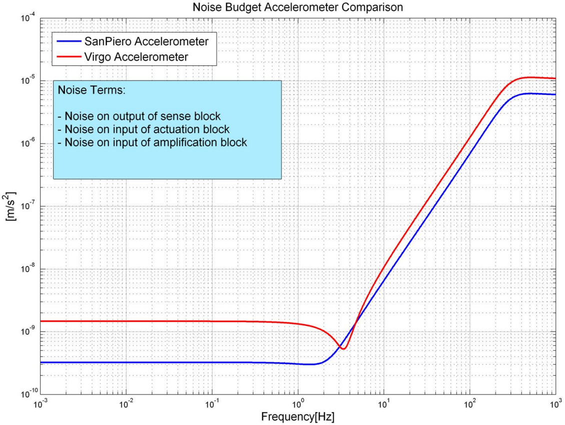
# Control Loop



# Accelerometer noise budget: single terms from the three noise sources on the system and their square sum



Noise budget of the two generations' accelerometer: comparison of the output sensitivity respect to the noise sources in the model



# Inverted Pendulum Model

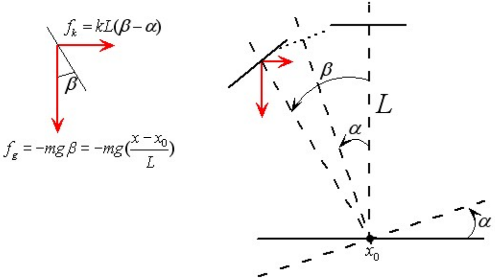
# Inverted Pendulum dynamic equations

*IP (translation)*:

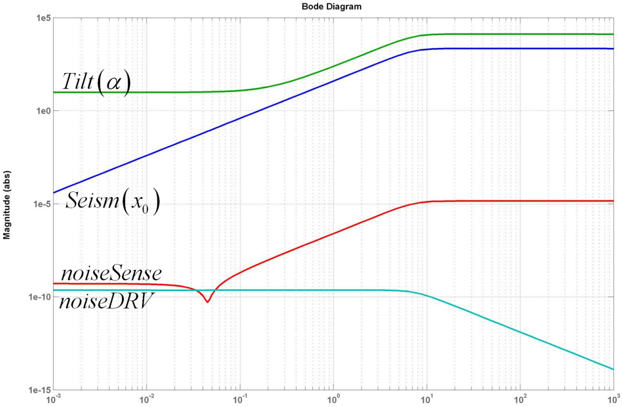
$$x_{x_0 IP} = \frac{\frac{s}{\omega_0 Q} + 1}{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1}$$

*IP (tilt)*:

$$x_{\alpha IP} = \frac{-L\left(1 + \frac{g}{m\omega_0^2 L}\right)}{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1}$$



## IP TFs respect to translation displacement $x$ and angular $a$ ($\omega_0 \cong 45mHz$, $Q \cong 10$)



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Accelerometer noise budget with exogenous inputs for seism and tilt models:

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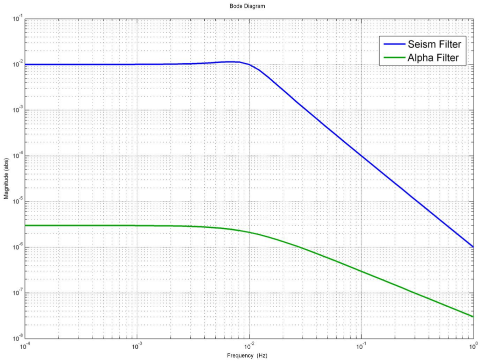
# Seism and tilt TFs

The accelerometer input signals for translation and tilt displacement are prefiltered through transfer functions of the approximated behaviour of the input position noises



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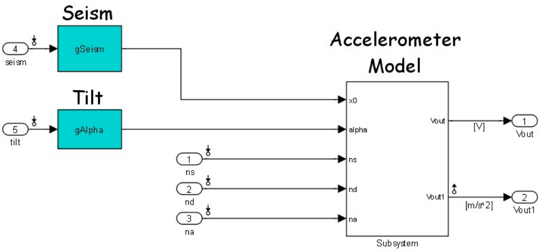
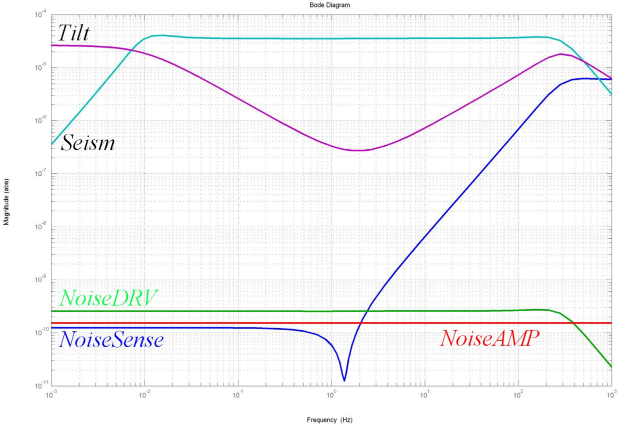


# Noise budget without IP



# Considerations

- Tilt effect dominates at low frequencies the accelerometer sensitivity
- After about $10mHz$ the sensitivity is dominated by the seismic translation noise

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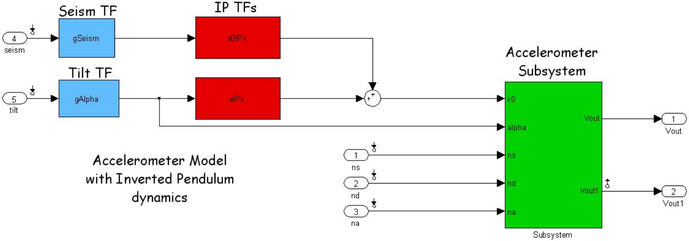
# Noise budget: IP added

The model is completed by the IP TFs:



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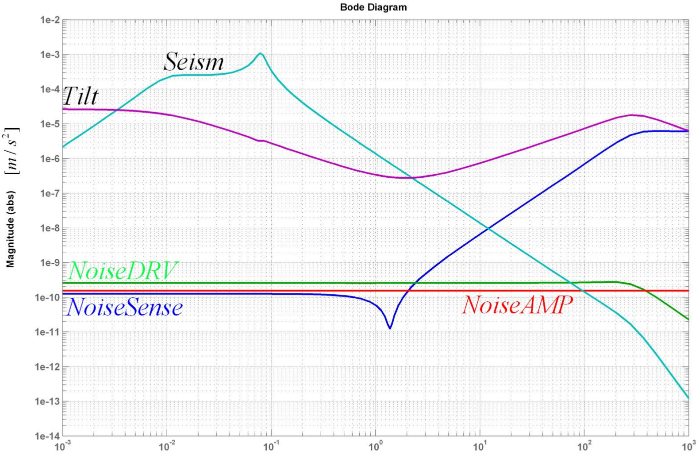

# IP "Accelerometer like" model

- As final analysis, the IP system has been treated as it was itself an accelerometer, in order to make a sensitivity comparison with the previous results
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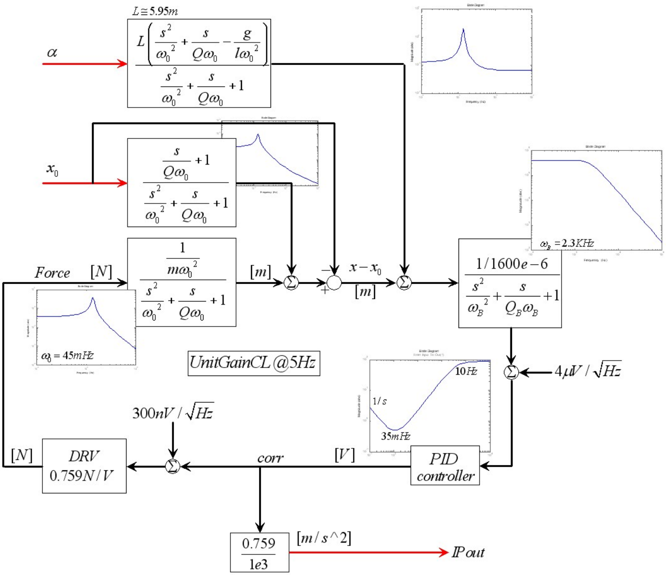
# Ip Model



$L \cong 5.95m$
$\alpha$
$\frac{L\left(\frac{s^2}{\omega_0^2}+\frac{s}{Q\omega_0}-\frac{g}{l\omega_0^2}\right)}{\frac{s^2}{\omega_0^2}+\frac{s}{Q\omega_0}+1}$
$x_0$
$\frac{\frac{s}{Q\omega_0}+1}{\frac{s^2}{\omega_0^2}+\frac{s}{Q\omega_0}+1}$
$Force$ $[N]$
$\frac{\frac{1}{m\omega_0^2}}{\frac{s^2}{\omega_0^2}+\frac{s}{Q\omega_0}+1}$
$[m]$
$x-x_0$
$[m]$
$\frac{1/1600e-6}{\frac{s^2}{\omega_B^2}+\frac{s}{Q_B\omega_B}+1}$
$\omega_B = 2.3KHz$
$\omega_0 = 45mHz$
$UnitGainCL @5Hz$
$4\mu V/\sqrt{Hz}$
10 Hz
1/s
35 mHz
$300nV/\sqrt{Hz}$
$[N]$
$DRV$ $0.759N/V$
corr
$[V]$
PID controller
$\frac{0.759}{1e3}$
$[m/s\wedge 2]$
IPout

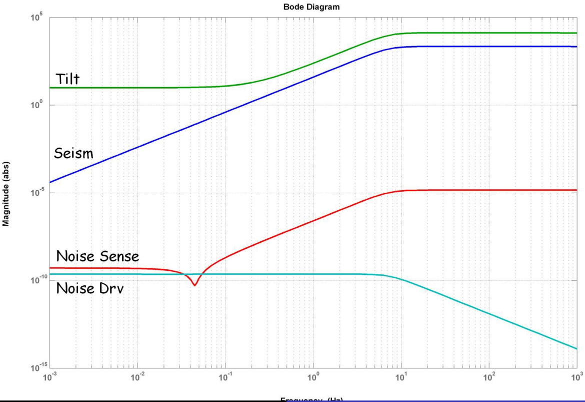
# IP Data

## IP Data

| | |
|---|---|
| $\omega_0$ | $\cong 45mHz$ |
| $mass$ | $\cong 1000Kg$ |
| $L$ (leg length) | $\cong 5.9485m$ |
| DC sense value | $625[V/m]$ |
| Unit CL gain | $@5Hz$ |
| DC TF value | $9.5e3[V/\mu]$ |
| DRV gain | $DCTFvalue \cdot m\omega_0^2 \cong 0.759[N/V]$ |
| Output conversion | $0.759e-3[ms^2/V]$ |
| Sense Noise | $\cong 4\mu/\sqrt{Hz}$ |
| Act Noise | $\cong 300nV/\sqrt{Hz}$ |

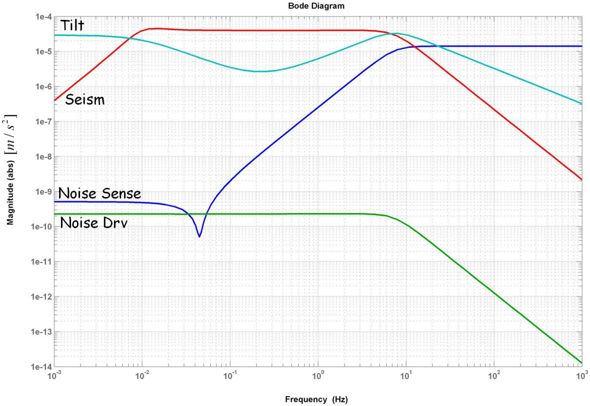
# IP TFs

IP TFs without the shape of the input signals, highlighting the sensitivity respect inputs and noise sources:

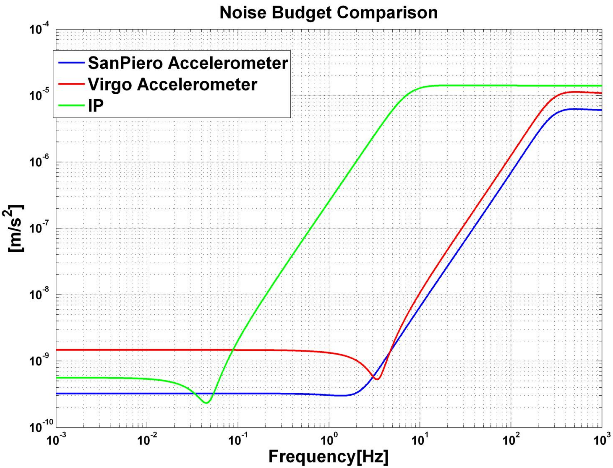


# Inverted Pendulum noise budget

IP noise budget with input filters for seism and tilt signals



# Sensitivity comparison with accelerometer and IP



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