

Unmodelled transient sources aka the Bursts

Generalities on short-duration GW signals. Searching for candidate events in data. Network analysis strategies. Upper limits, detection confidence and all that

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VESF DA School – Spring 2011

GW vade mecum

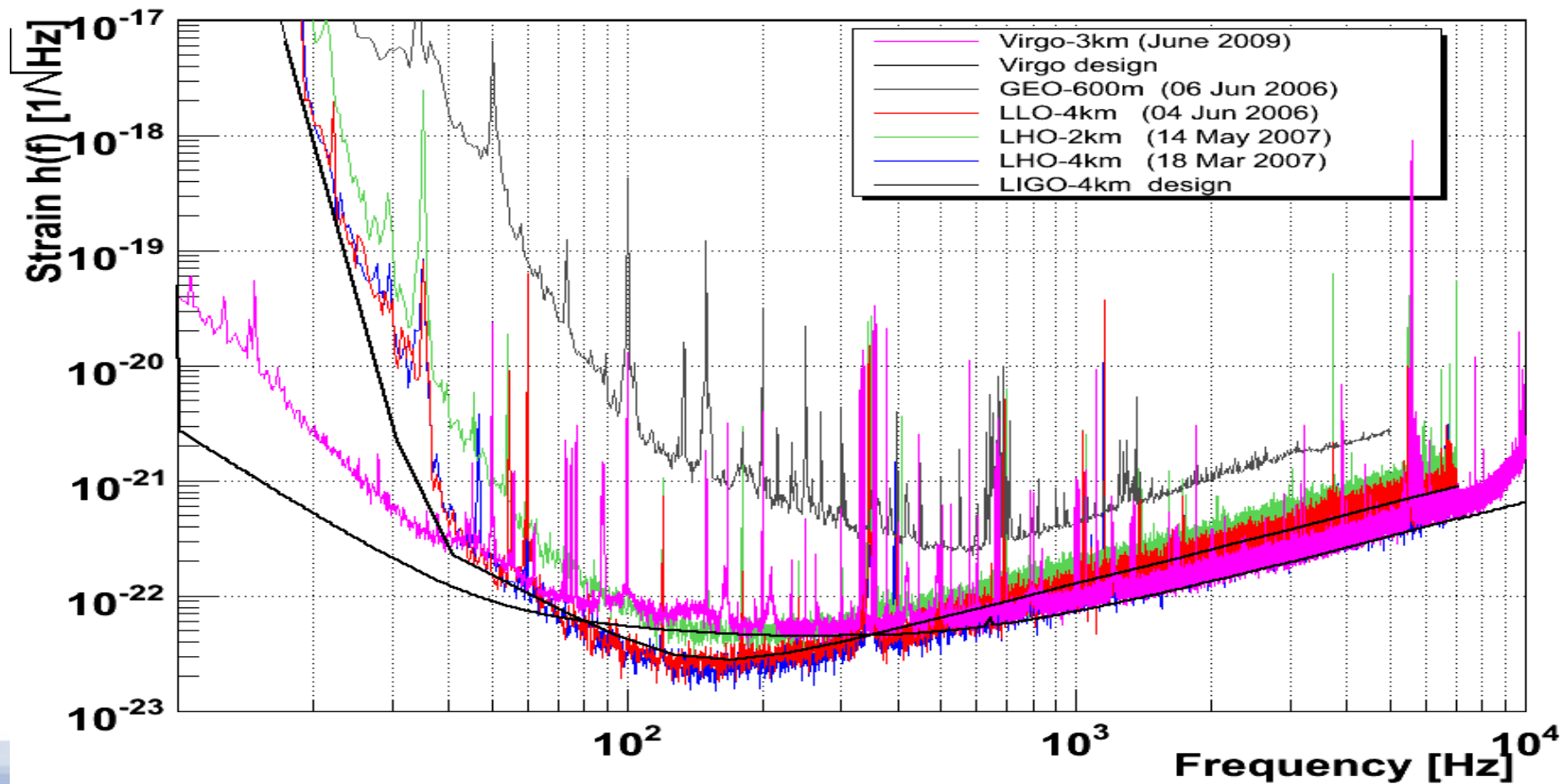
- GW emission from quadrupole momentum
- 2 polarizations : +, x
- Propagating at celerity c
- Power emitted:

$$P = \frac{G}{c^5} \epsilon^2 v^6 M^2 R^4 \quad \text{or} \quad P = \frac{c^5}{G} \epsilon^2 \left(\frac{v}{c}\right)^6 \left(\frac{R_s}{R}\right)^2$$

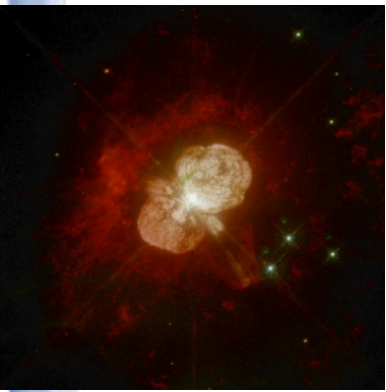
- Compact object
- Relativistic motion
- Large asymmetry

GW vade mecum

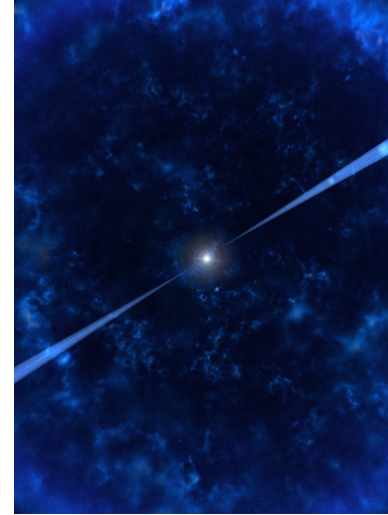
- We will only focalize on:
 - Ground based systems
 - Large band detector – Michelson interferometer
 - Frequency band : 50 Hz -> 5 kHz



Transients sources



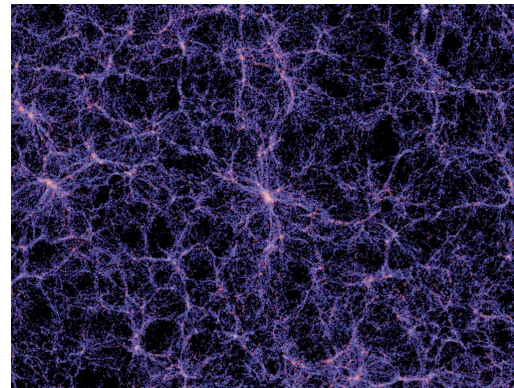
Supernovae
(asymmetric core
bounce)



**Pulsars/Neutrons
stars**
(instabilities)



**Compact binary
coalescence of neutron
stars &/or black holes**



Cosmic strings

The unexpected



Bursts : Let's define the beast

- All transient signals with a duration < 1 second
- Recover a large variety of type of sources :
 - most of the time : unmodelled sources
 - Few of them : simples models exist
 - Numerical simulations give plenty of waveforms
- Easily mimic by glitches in the detector
- Need robust analysis
- Using a network will help a lot !

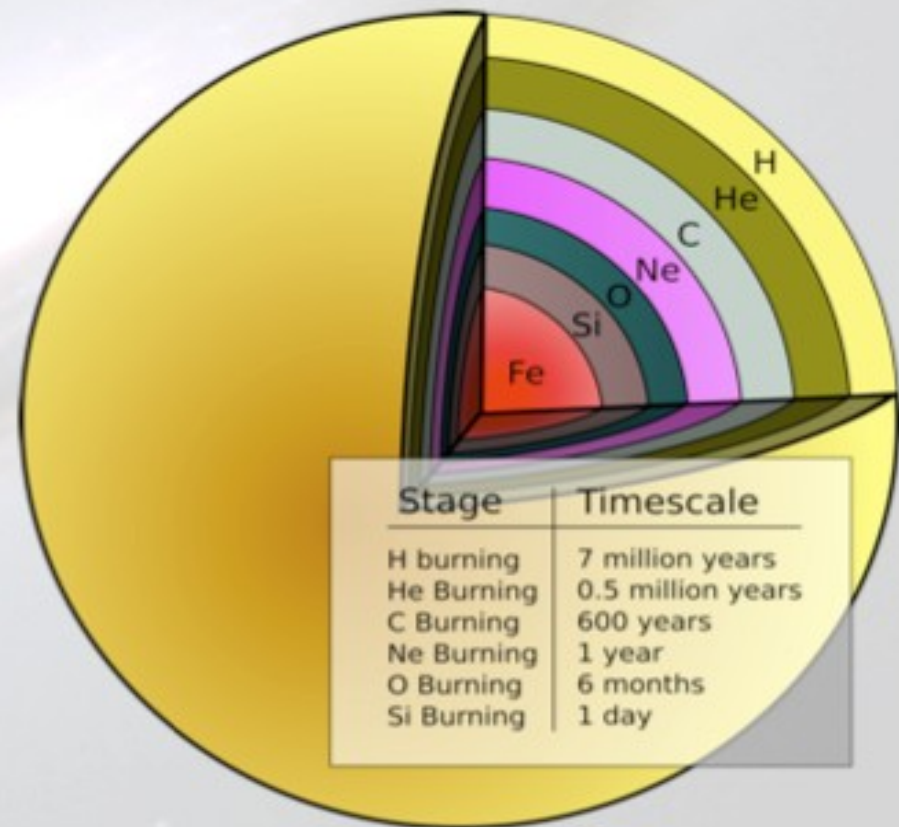
Sources

Sources

- A gravitational wave source needs to be
 - compact
 - asymmetric
 - relativistic
- What type of source
 - Collapsars and supernovae
 - Black holes
 - GRB, SGR, pulsars glitches
 - Exotic physics

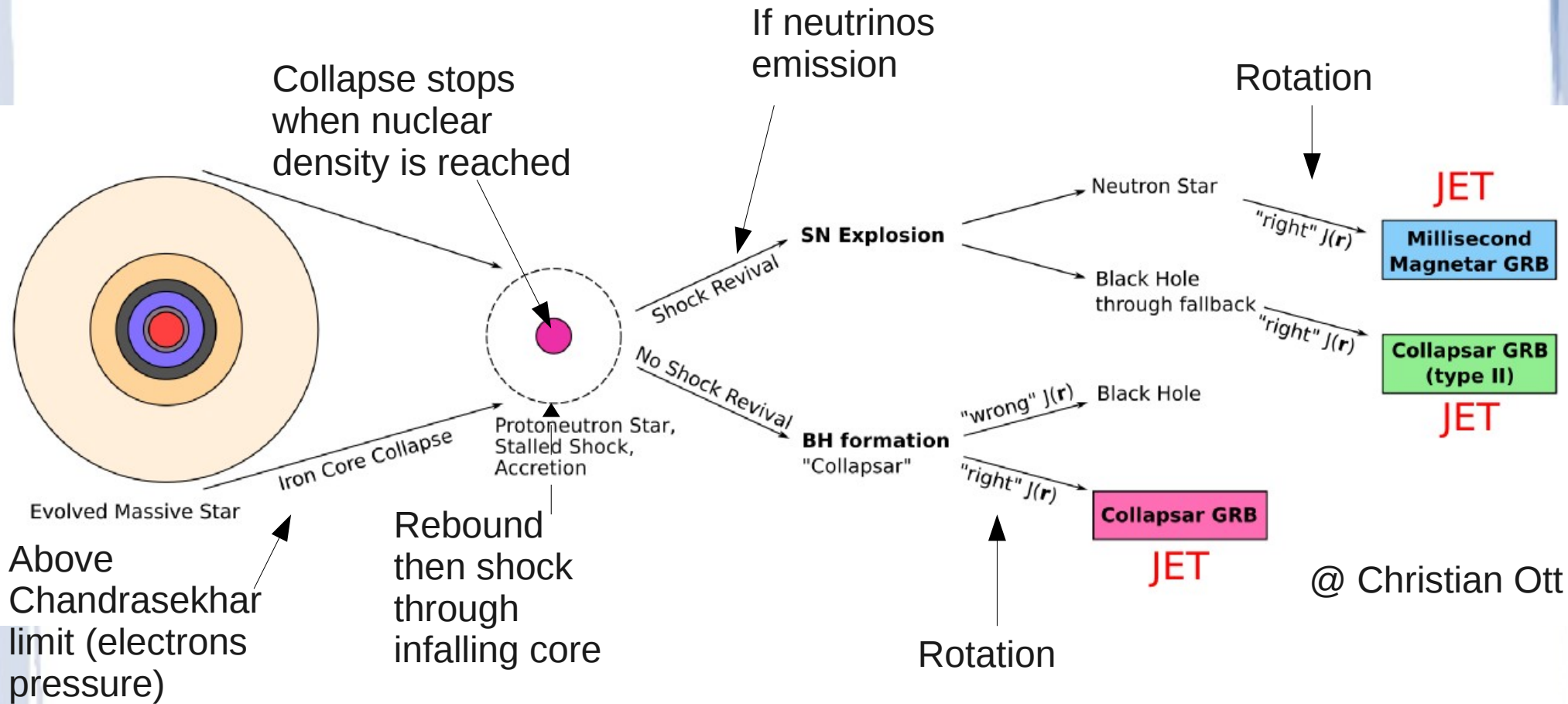
Stellar Collapse: Building the Core

- Stars spend most of their lives burning hydrogen.
- The product – Helium – settles in the core and will burn when temperatures increase sufficiently.
- For massive stars ($M > 8-10M_{\text{sun}}$), the process continues through Carbon, Oxygen, ... , up to Iron.
- This process does not continue past Iron as Iron is one of the most tightly bound nuclei.
- Iron core builds up in center of star.



A. C. Phillips, *The Physics of Stars*, 2nd Edition (Wiley, 1999).

Stellar collapse in a nutshell



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Supernovae Type

Classification of Supernovae

	Type Ia	Type Ib	Type Ic	Type II	AIC
Spectrum	<p>No Hydrogen</p> <p>Silicon</p>	<p>No Hydrogen</p> <p>No Silicon</p> <p>Helium</p>	<p>No Hydrogen</p> <p>No Silicon</p> <p>No Helium</p>	<p>Hydrogen</p>	<p>Unknown</p>
Mechanism	<p>Thermonuclear Explosion</p>	<p>Core Collapse</p>			
Lightcurve (powered by)	<p>Generic (Radioactive Decay)</p>	<p>Strong Variations (Shock Ionization/Recombination; Radioactive Decay at Late Times)</p>			<p>Unknown (Like Ia?)</p>
Neutrinos	<p>Insignificant</p>	<p>ca. 100 x Visible Energy</p>			
Compact Remnant	<p>None</p>	<p>Neutron Star, Black Hole (?)</p>			<p>Neutron Star</p>

↑
Not for us !

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The hard life of simulations

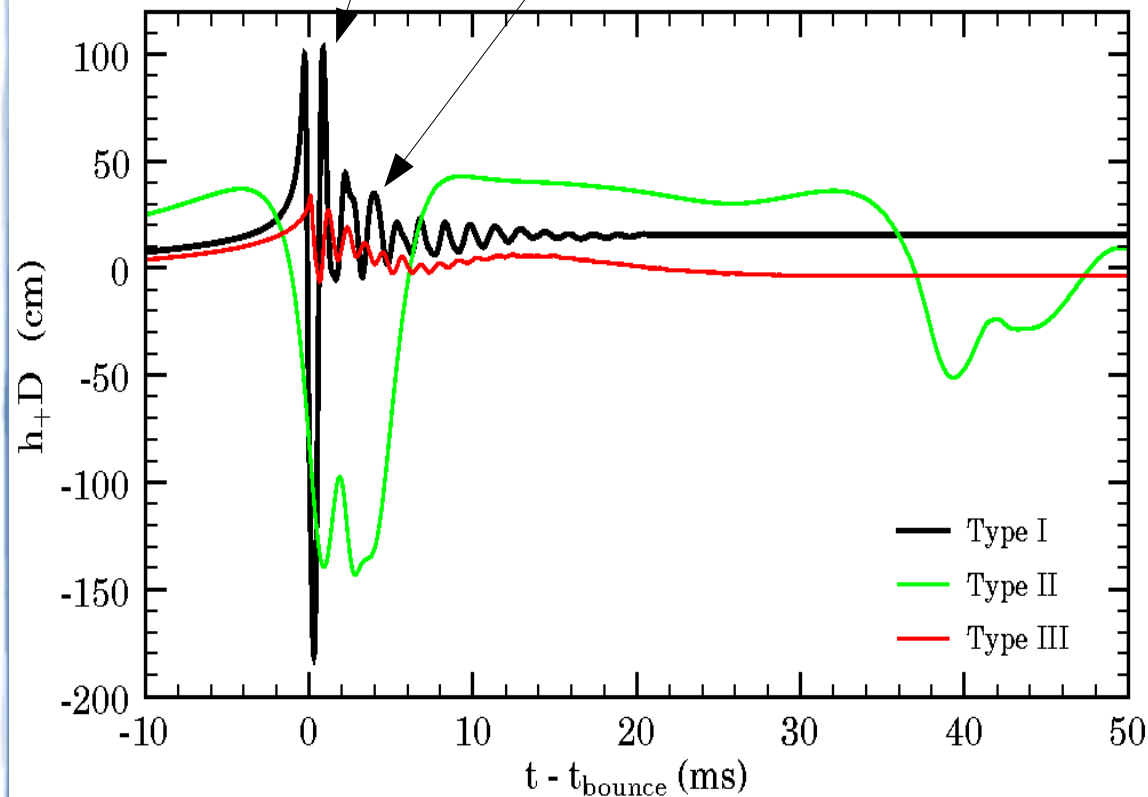
- Different problems on simulation side :
 - How to simulate such complicate objects ? - problem with resolution and dimension (and computation problems)
 - Which type of physics :
 - Full general relativity and/or approximation
 - microphysics : neutrinos, convection, instabilities, MHD, ...
 - Equation of state of nuclear matter
 - How to light a supernovae ?
 - How much spin on the initial star ?

Non linear
equations

Large uncertainties on the waveforms

Collapsar GW emission

- Simple case : 2D axisymmetric
 - Generic features :
 - Peak for the core bounce
 - Ring down of the central object

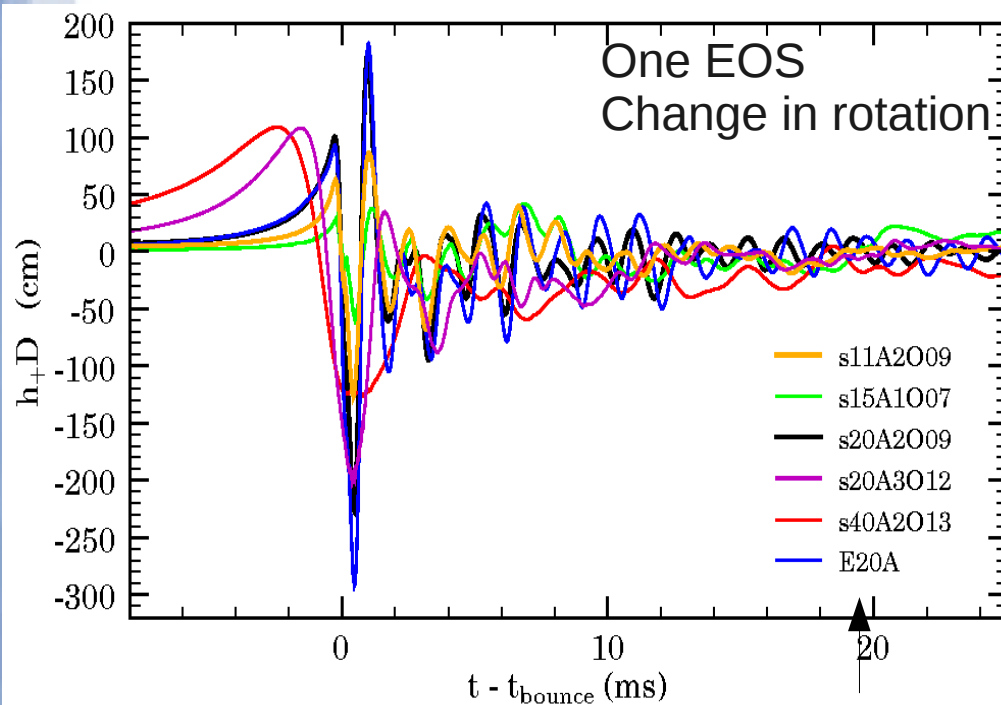


- However waveforms could change a lot
- Important parameters:
 - Equation of state
 - spin

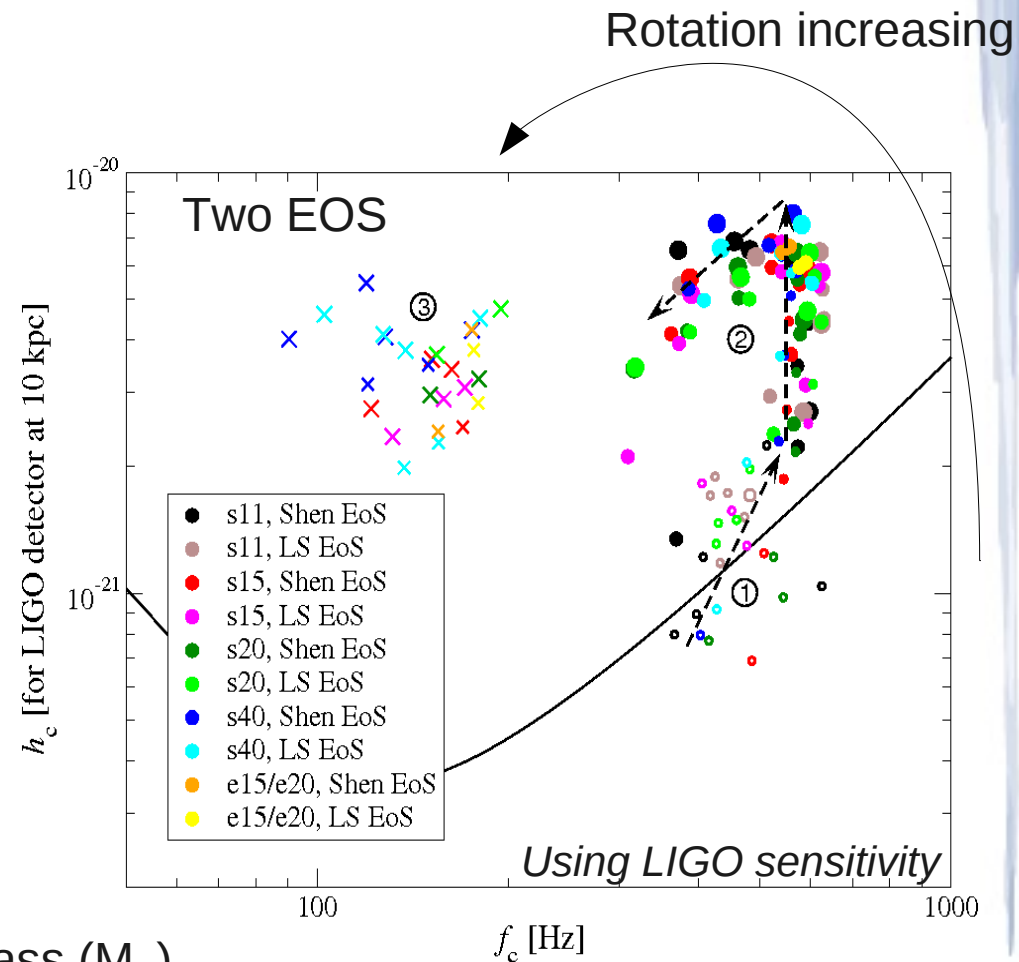
Simulations from Dimmelmeier et al

Collapsar GW emission

- Last generation of simulation
- Usual shape
- convergence on the results

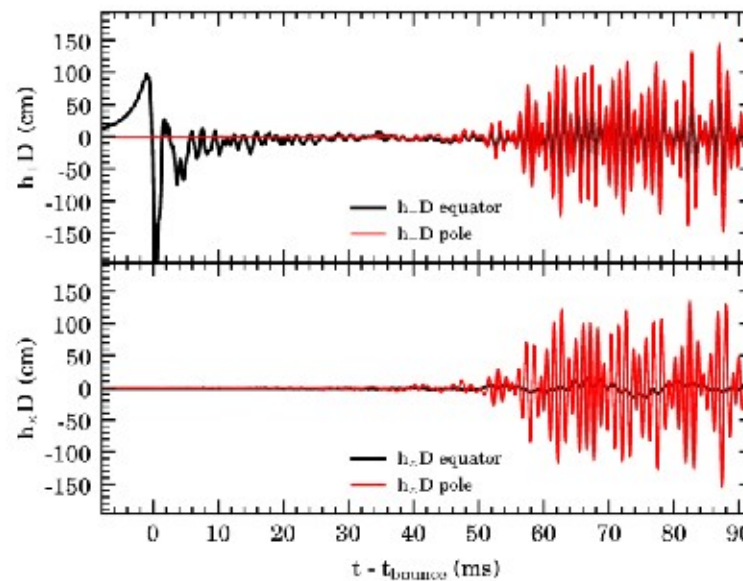
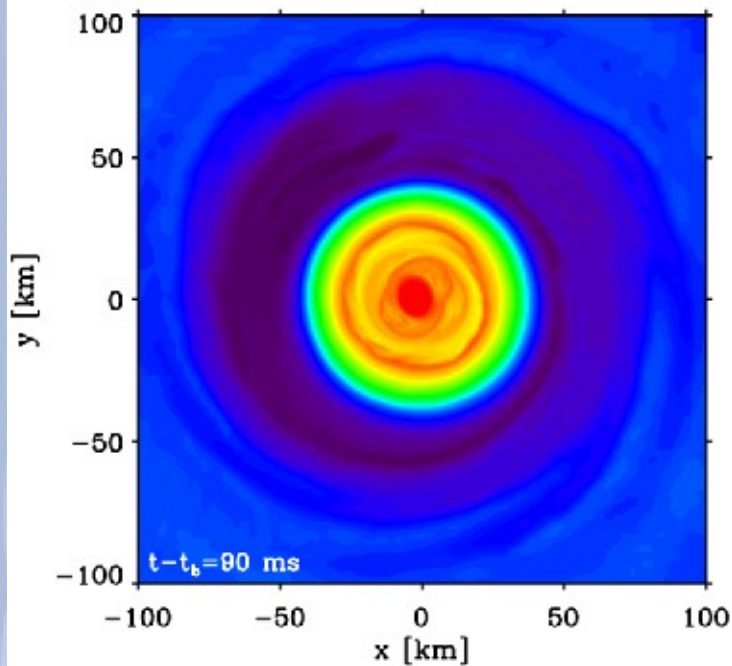


Object mass (M_{\odot})



Collapsar GW emission

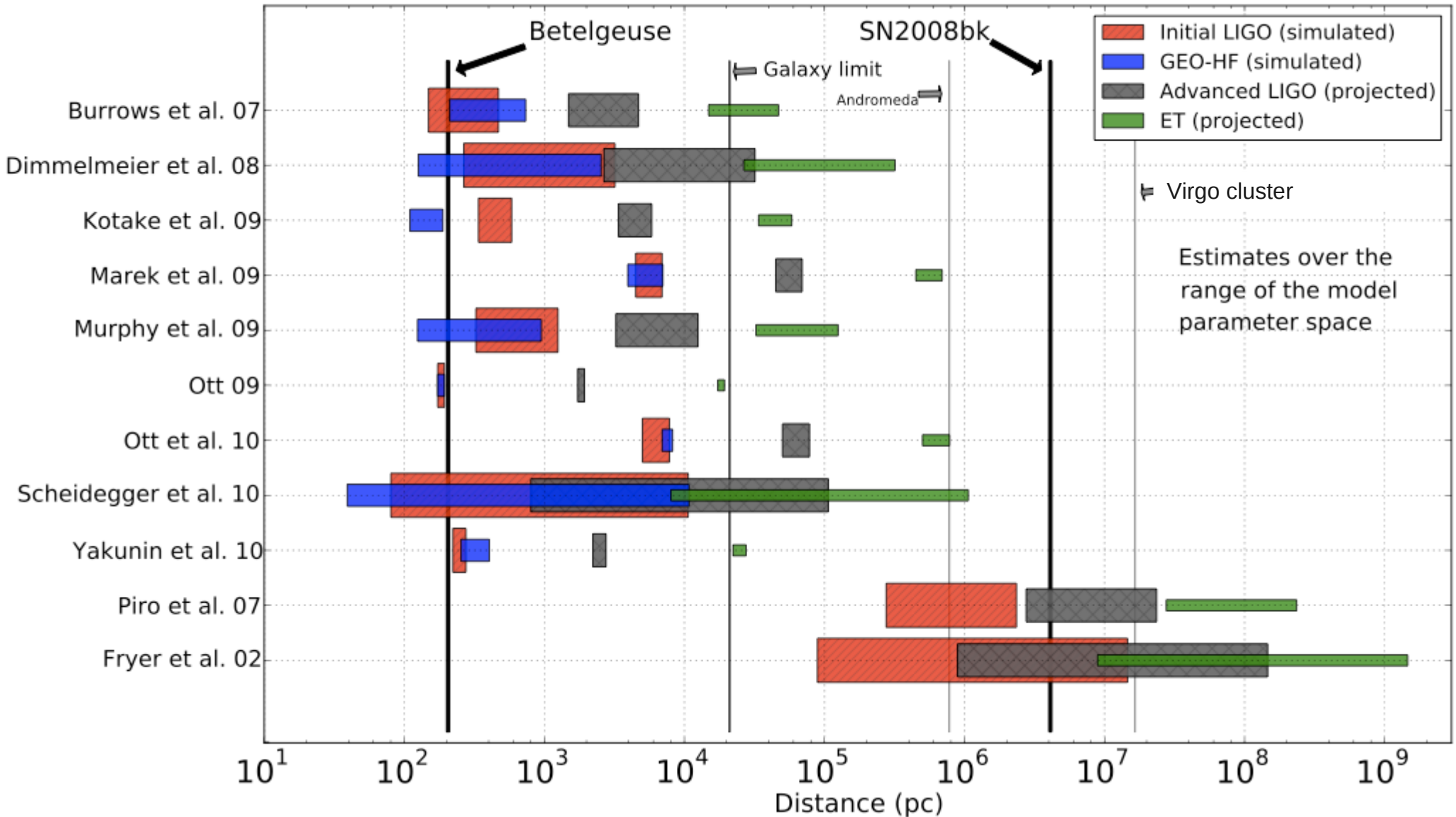
- Creation of bar mode
- More than the bounce – acoustic mechanism
 - Require excitation of large amplitude proto neutron star by turbulence and instabilities



How far can we go ?

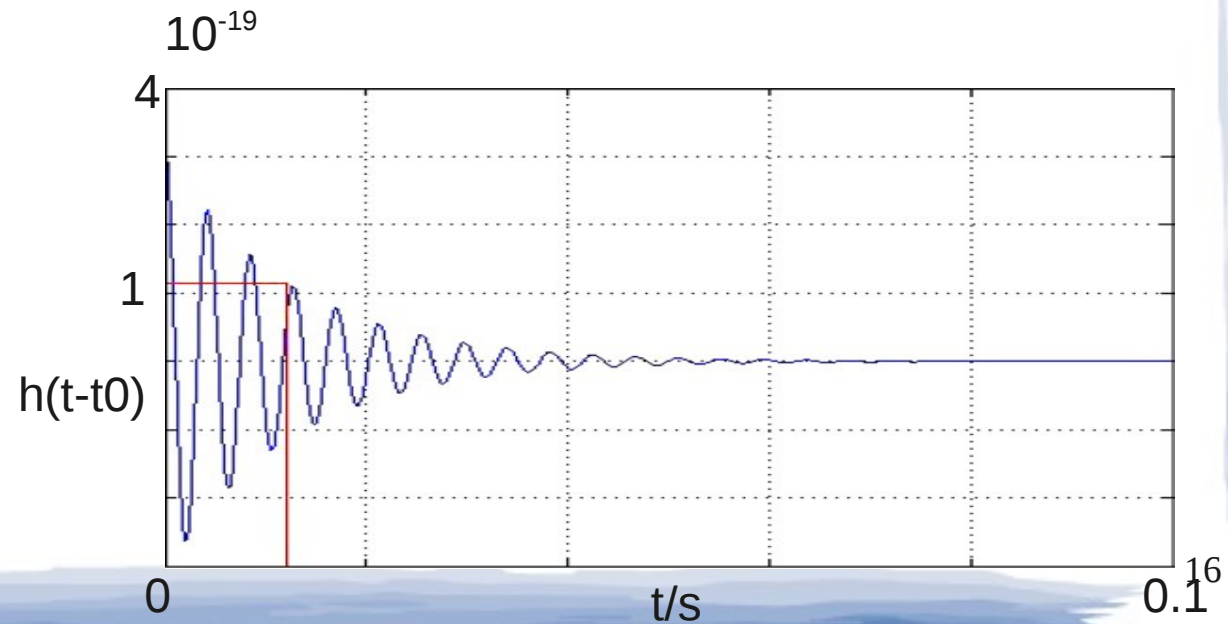
Events rate : 3-5 / galaxy/ century

Preliminary reach estimates



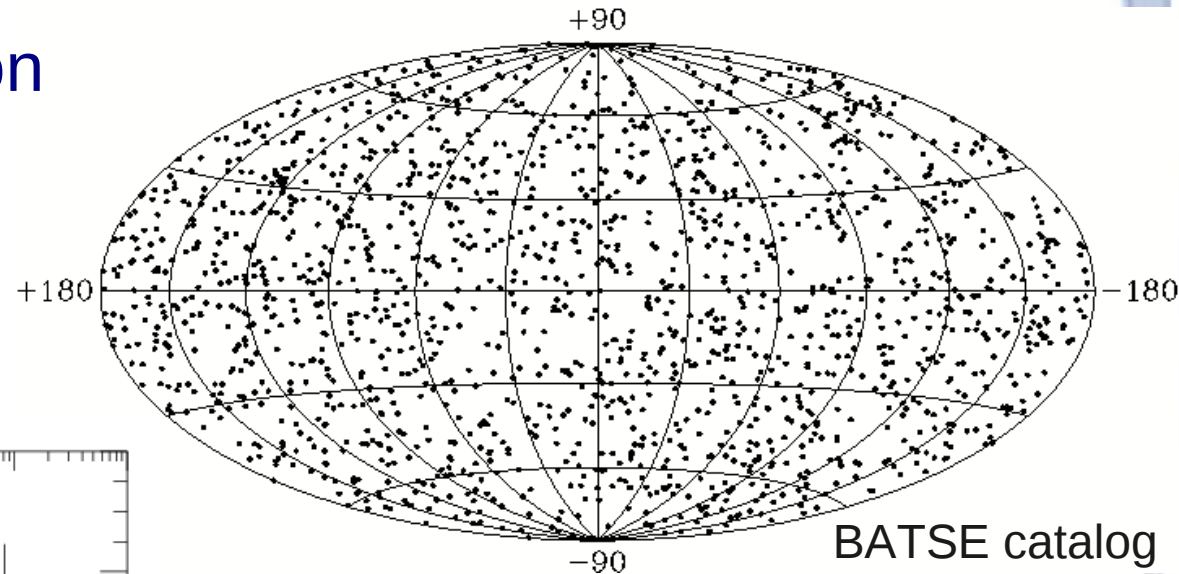
Black hole

- When created or excited, black hole will lost energy through GW emission
- Typical waveform : damped sine
- Important parameter : Mass
- Only way to detect directly such object

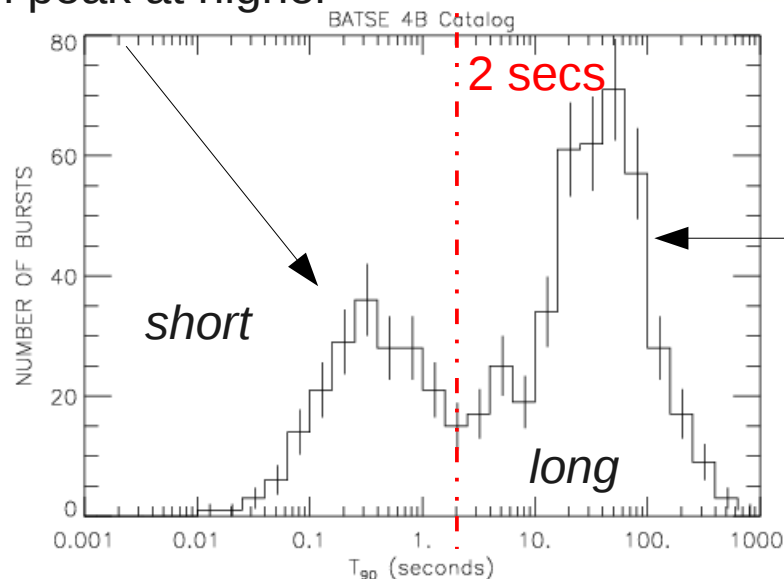


Gamma-ray bursts

- Find in the 70's : burst of gamma-rays
- Isotropic sky distribution
- 2 populations



Spectrum peak at higher energy



Spectrum peak at lower energy

T90 : duration of 90 % of photons counts

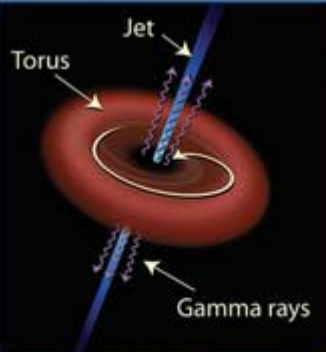
Gamma-ray bursts

Gamma-Ray Bursts (GRBs): The Long and Short of It

Long gamma-ray burst (>2 seconds' duration)



...becoming so dense that it expels its outer layers in a supernova explosion.



Short gamma-ray burst (<2 seconds' duration)

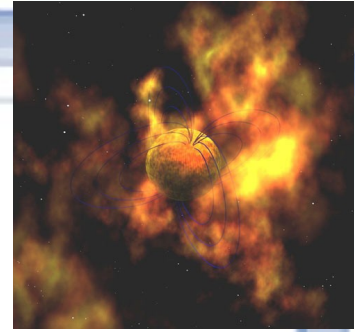


...eventually colliding.

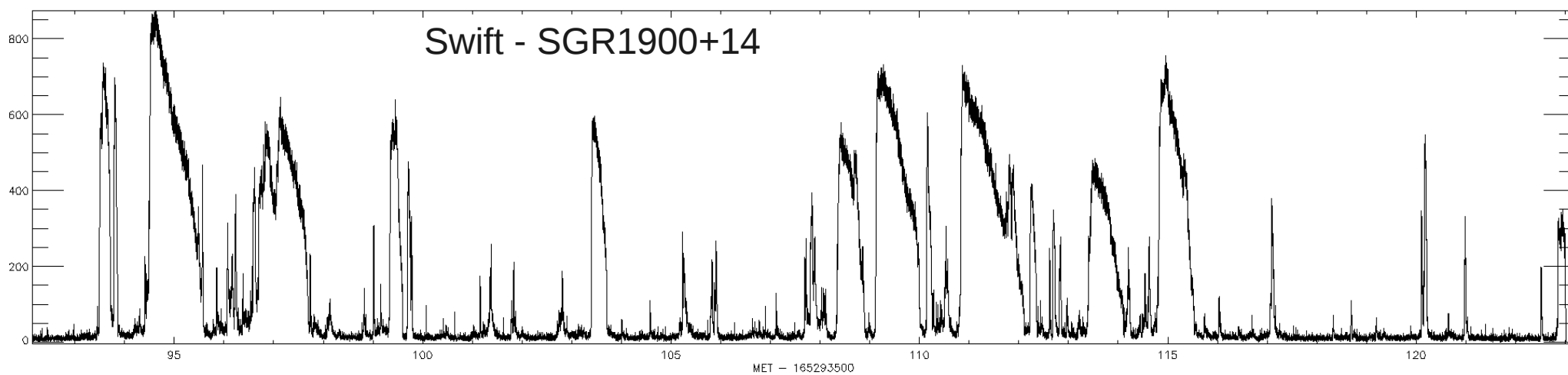


- Long GRB :
 - Massive rapidly spinning star collapse or explosion (hypernovae)
- Short GRB :
 - Coalescence of compact objects
- Gamma emission :
 - 10^{51} erg $\sim 10^{-3} M_{\odot} c^2$
- Distance : \sim Gpc
- $E_{\text{GW}} \approx 10^{-2} M_{\odot} c^2$

Soft gamma rays repeater

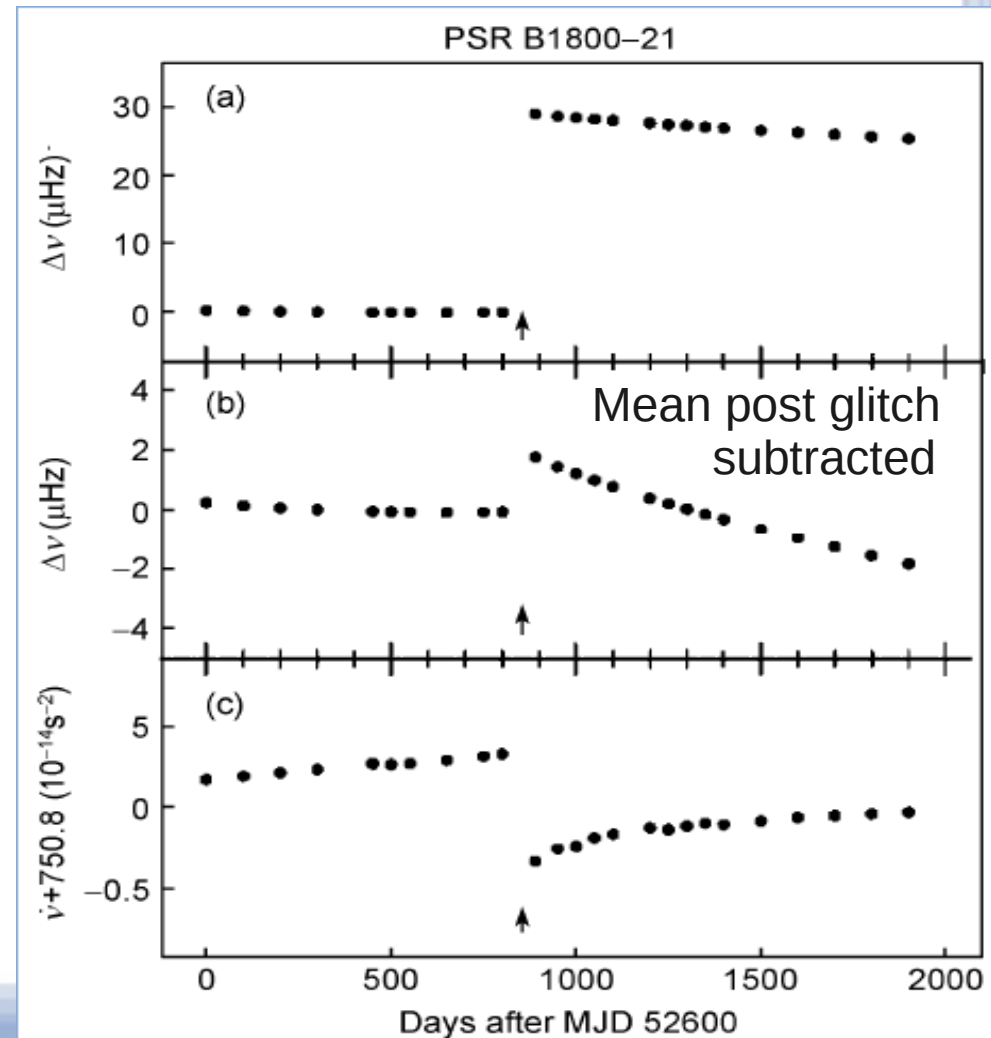


- Emit flares of soft gamma rays 10^{42} -> 10^{46} erg
- Certainly magnetar (neutron star with $B > 10^{15}$ G)
- Could be related with cracking of the crust
 - Possible excitation of vibrational modes
 - May couple to GW



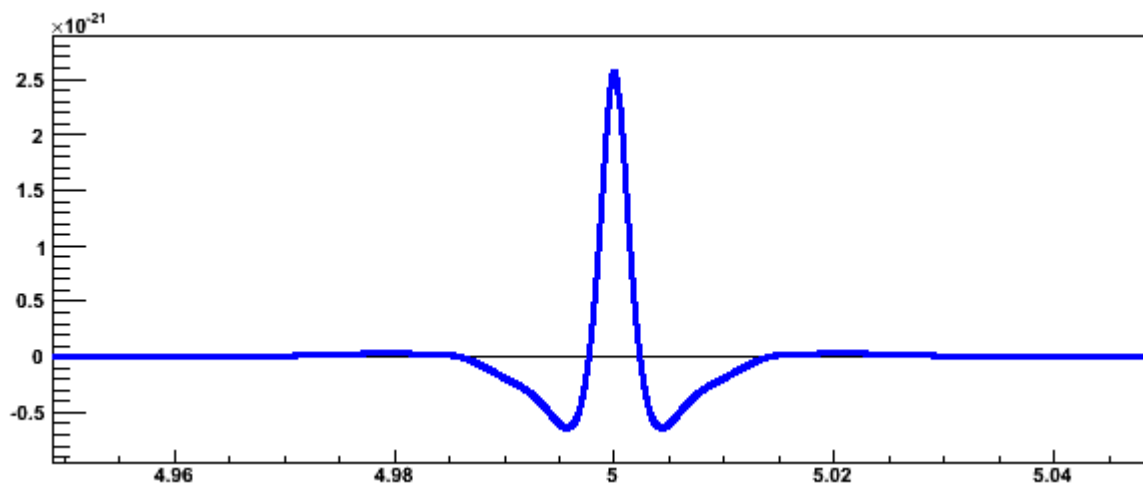
Pulsar/Neutrons star glitches

- Some pulsars exhibits glitches in pulsating frequency
- Mechanism not clear but
 - Could be also crust cracking
 - Differential rotation between core and crust
- Could coupled to GW



Exotic physics (?)

- Cosmic strings : topological defect 1D – formed when having a phase transition in the primordial universe
- If 2 parts of the string meet, possible reconnection
 - Loop -> oscillation -> cusps - GW emission
- Will emit GW with a known-signal : power law in frequency : $f^{-4/3}$

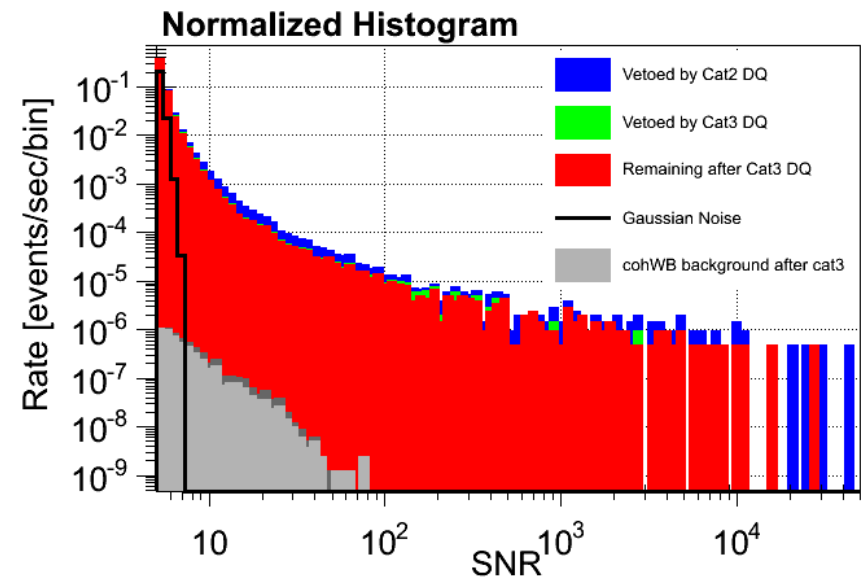
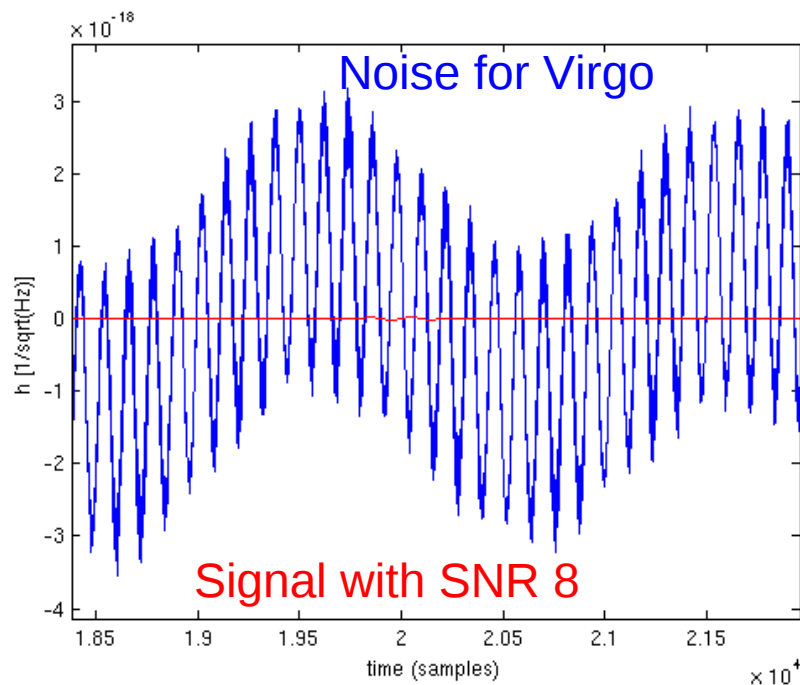


GW signal from a string cusp

Searching for bursts

Searching for Bursts signals

- We try to find weak signals
- For most of the cases we do not know the waveform
- Noise is dominant on the output of the detector



Linear filtering

- Generic form : correlation between data stream and a kernel

$$\int h(\tau) \phi(t - \tau) d\tau$$

- Wiener worked on recovering a known signal u mixed with gaussian noise $n(t)$, best solution is :

$$\text{(Fourier Transform)} \quad \tilde{\phi}(f) = \frac{\tilde{u}(f)}{S_n(f)}$$

Power spectral density

- Optimal visibility is given by SNR:

$$\rho^2 = \int \frac{|\tilde{u}(f)|^2}{S_n(f)} df$$

The match filtering technique

- Next step : using a template (exact waveform, an approximation, a simple shape, ...) and slid it on the data

$$c(t) = \frac{\langle s(t), d(t) \rangle}{\|s\|} \quad \langle s(t), d(t) \rangle = \frac{\int \tilde{s}(f) \tilde{d}^*(f) df}{S_n(f)} \quad \|s\| = \max_t \langle s(t), s(t) \rangle^{1/2}$$

- We can only run on a limited number of templates
 - You need to tile your parameters space by computing matching between 2 templates

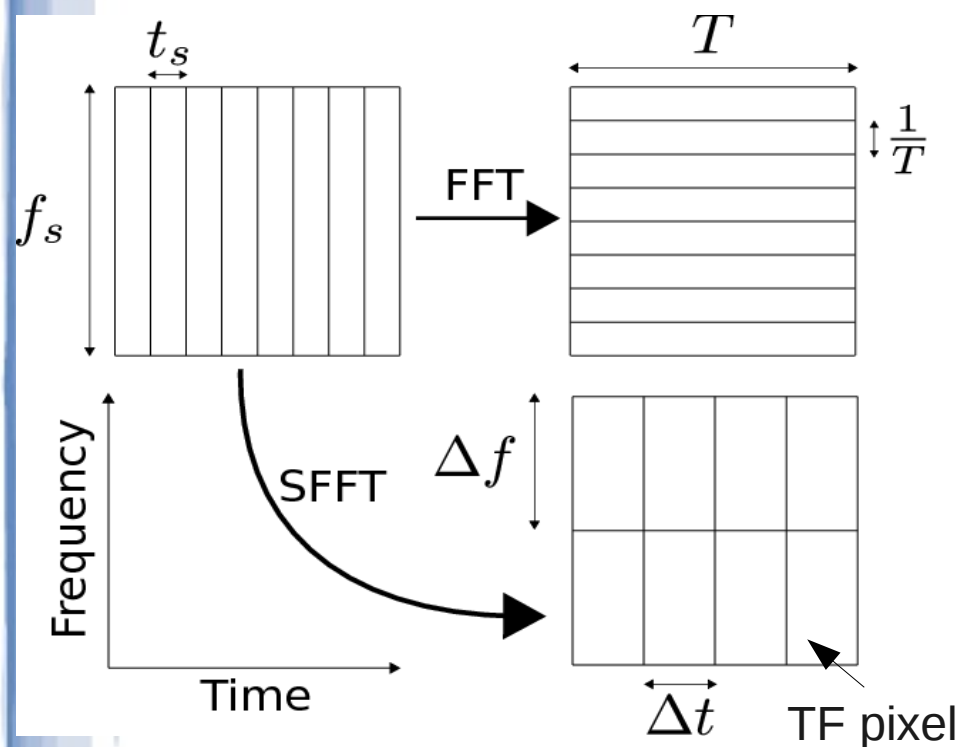
$$\Gamma(\psi, \phi) = \max_t \langle \phi(t), \psi(t) \rangle$$

example with Gaussians :

$$\sigma_k = (1 + 2\sqrt{\Gamma})^{k-1} \sigma_{min} \quad \forall k \in \mathbb{N}^*$$

Time-frequency

- Using time and frequency information on time series could help a lot
- Most wide-known example : Fourier transform
- One limitation : the Heisenberg-Gabor incertitude



$$\sigma_t \sigma_f \geq \frac{1}{4\pi}$$

$$\sigma_t = \int (t - t_0)^2 |s(t)|^2 dt \text{ with } t_0 = \int t |s(t)|^2 dt$$

$$\sigma_f = \int (f - f_0)^2 |\tilde{s}(f)|^2 df \text{ with } f_0 = \int f |\tilde{s}(f)|^2 df$$

Fourier Transform

- We have data sampled \rightarrow we cannot have info above Nyquist frequency ($f_{\text{sampling}}/2$)
- Discrete Fourier transform:
 - Naive implementation : $O(N^2)$
 - FFT implementation : $O(N \log N)$
- FFT on non periodic signal \rightarrow power leakage
 - Need to window your data
- Need to have 2^N samples for FFT
 - Perform zero-padding

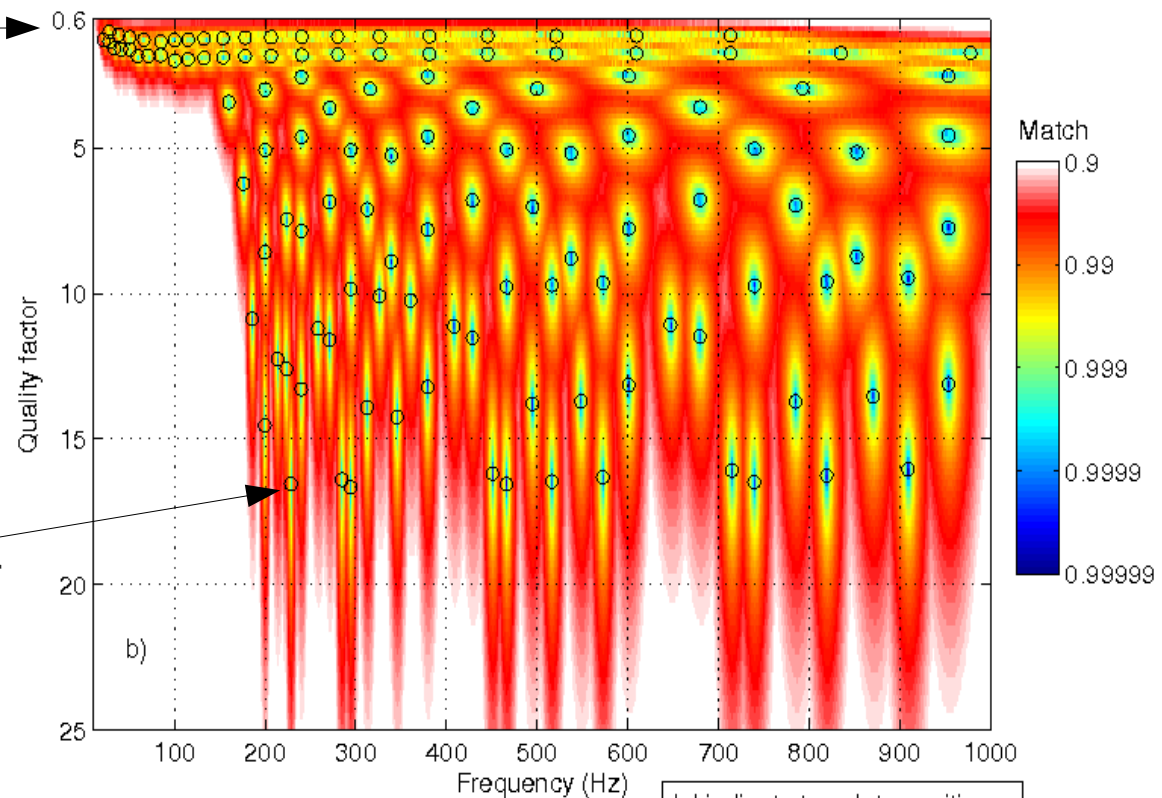
*For more information, have a look
To any Numerical Recipes book*

Bank of templates

- Using sine/cosine Gaussian type signal:
 - Allow to cover the full time-frequency plane
 - Well define in frequency : f
 - Allow to fix template length : $Q = \sigma \times 2\pi f$

Short duration signals

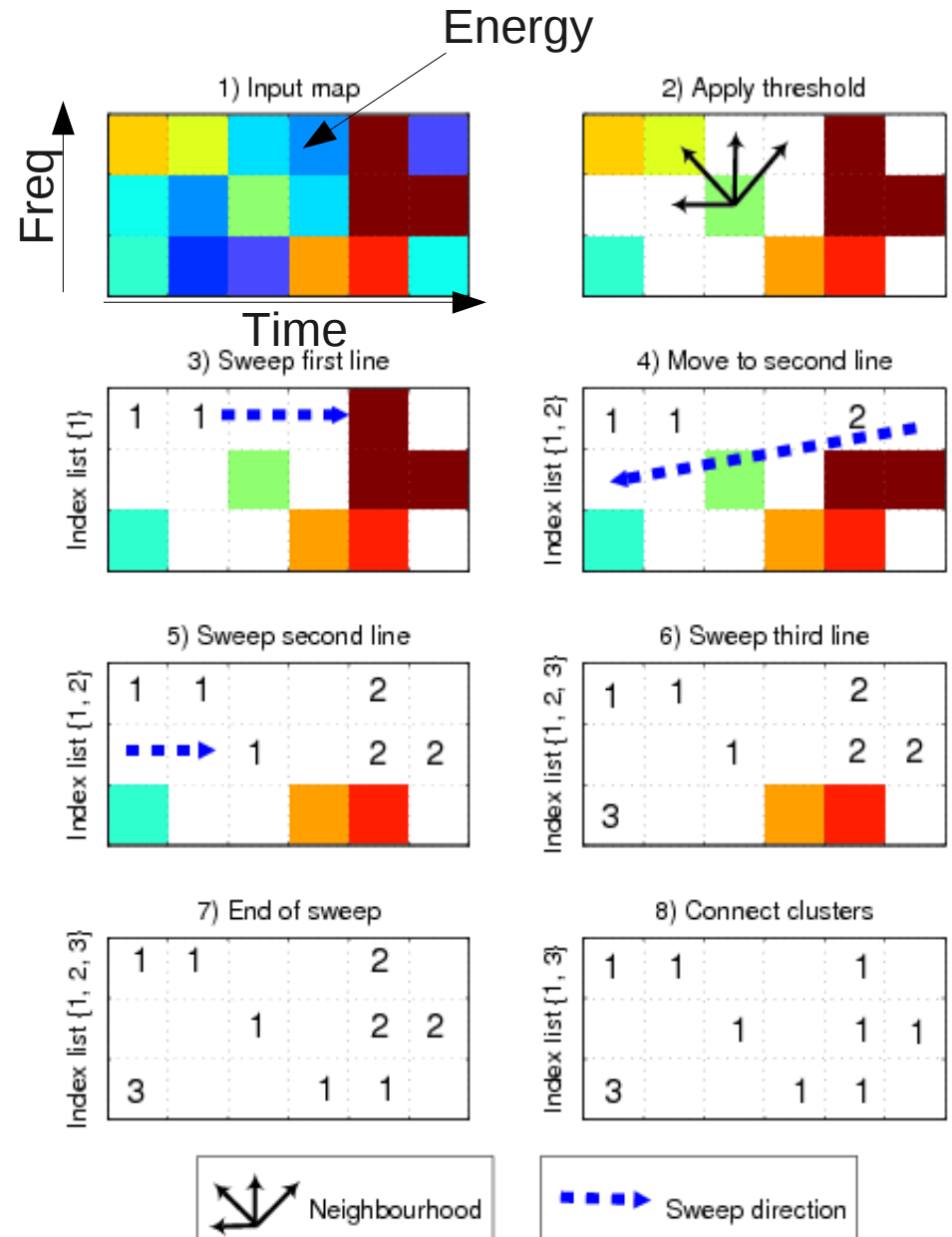
Limit on the number of templates



Clustering

- Now we need to define the notion of event
 - Time
 - Frequency
 - Energy

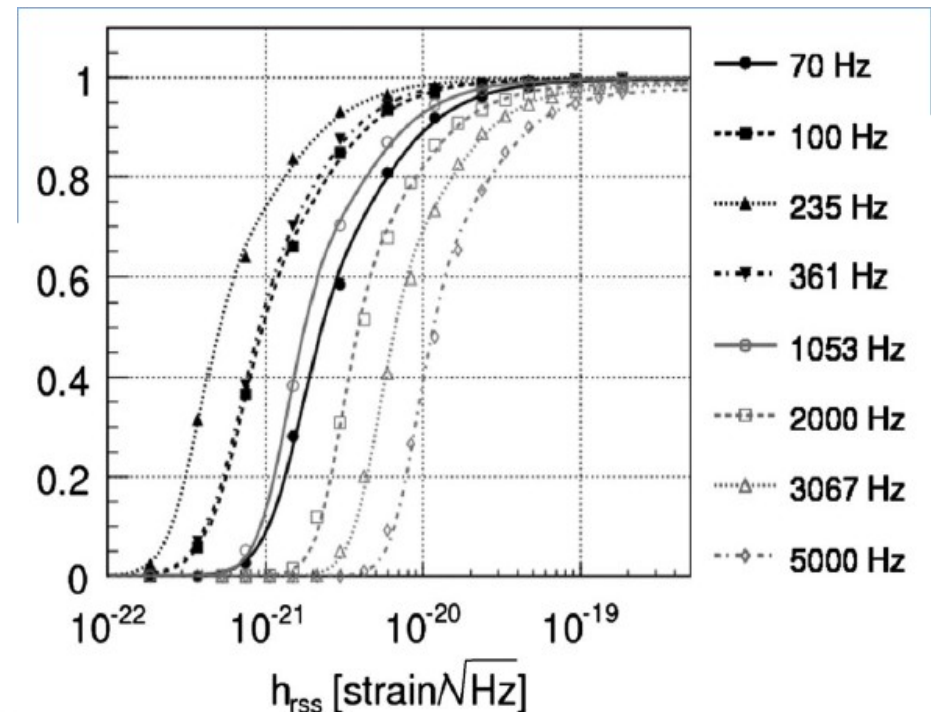
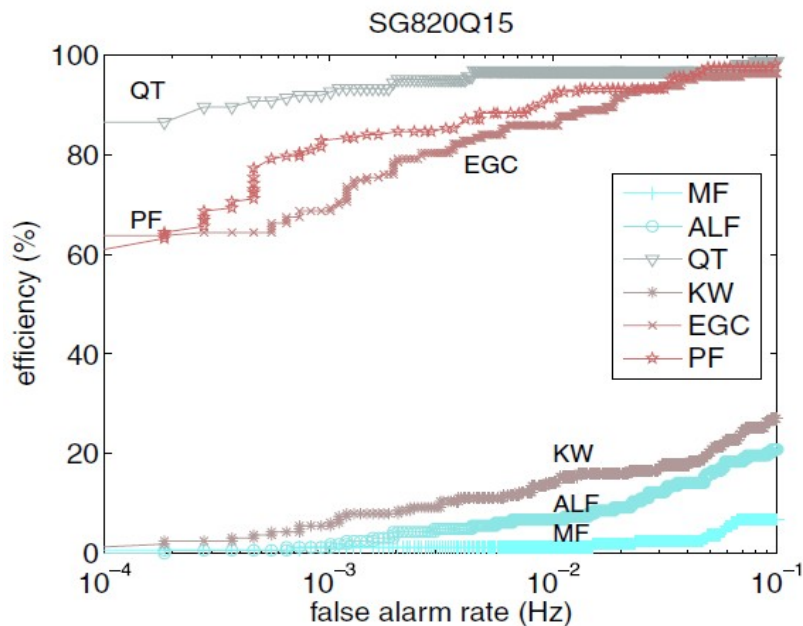
- We will rearrange the TF map in clusters of pixels
 - Thresholding : remove “noise only” pixels
 - Find joint pixels above threshold
 - Merge joint clusters
 - Threshold on the cluster energy



Compute efficiency

- How to compare efficiency between algorithms
- One possible tool : Receiver Operating Characteristic (ROC) : efficiency vs false alarm probability
- Look also to efficiency vs strain GW

$$h_{rss} = \sqrt{\int |h_+|^2 + |h_x|^2 dt}$$



Single ITF output

- Now is time to apply your Data Quality flags
- Continue to clean as much as possible your tails
- Use external triggers
- Do you trust enough your triggers distribution ?
- Put an upper limit ...

- Or use others instruments to improve our analysis

Network analysis

- Beam pattern
- Coincident vs coherent
- Sky localization

What is available on the market ?

Only for large band detectors

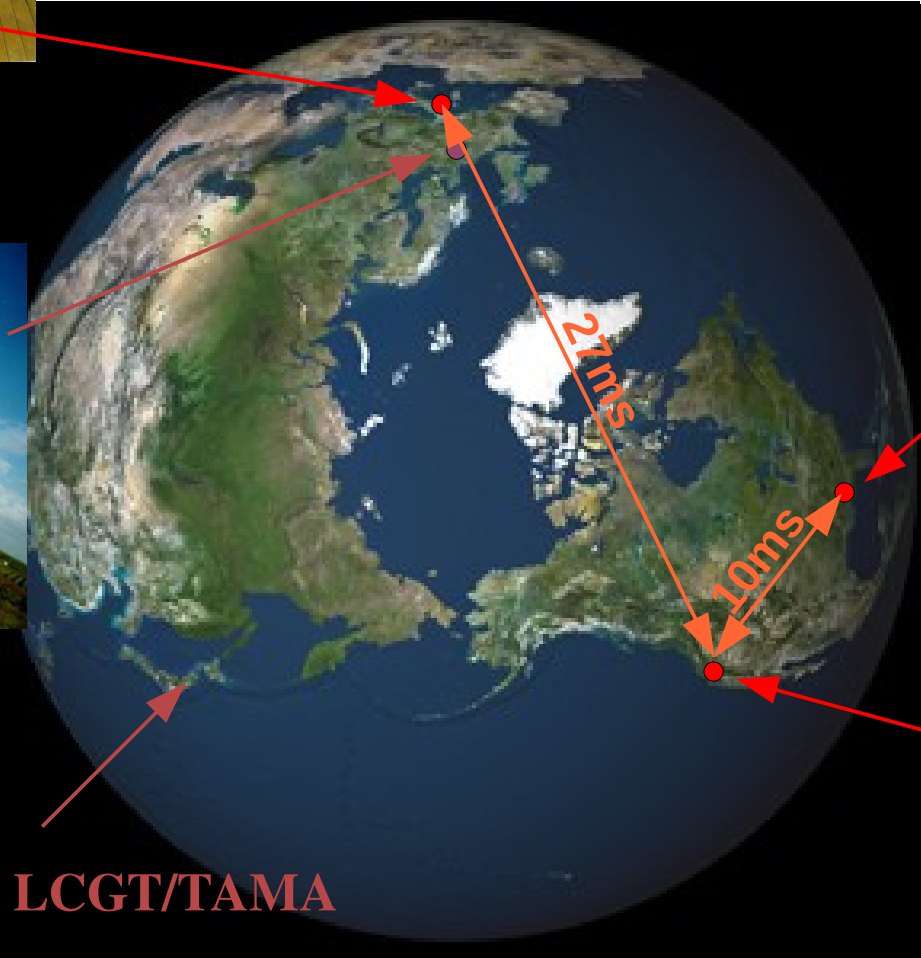
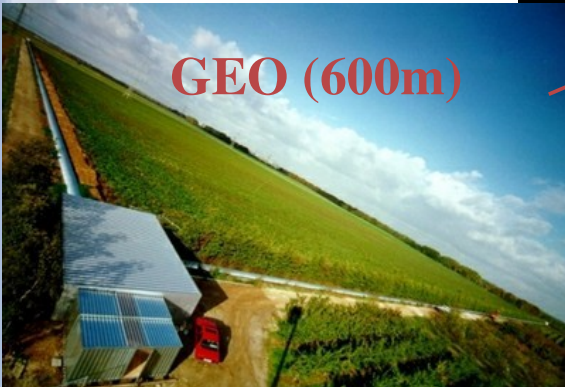
Virgo (3 km)



Livingston (4 km)



GEO (600m)



Hanford (4&2 km)



LCGT/TAMA

What do we have ?

- We will focus on the LIGO-Virgo network
 - H1L1 : 10 ms
 - H1V1 : 27 ms
 - L1V1 : 26 ms

- Signal in each ITF:

$$d_i(t) = F_{+,i}(t)h_+(t) + F_{x,i}(t)h_x(t) + n_i(t)$$

- Noise is (most of the time) independent from one site to the other
- Strength of the GW signal will depend of the beam pattern and the relative sensitivity

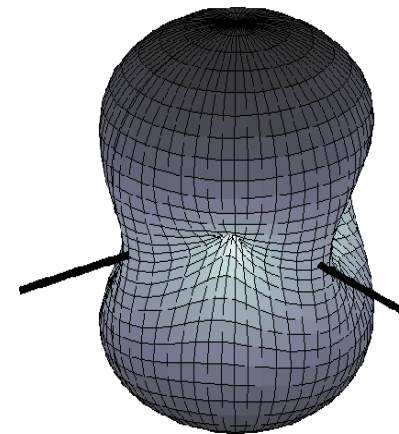
Beam pattern : we do not point into the sky !

- Beam pattern will depend on the position and orientation of the arms of the detector
 - Maximum of sensitivity if GW arrives by zenith (or nadir)
 - Null sensitivity if coming through the bisectrix
- Dependence also on the polarization of the source
- Due to earth rotation we can scan the sky

$$F_+ = \frac{1}{2} (1 + \cos^2(\Theta)) \cos(2\Phi) \cos(2\Psi) - \cos(\Theta) \sin(2\Phi) \sin(2\Psi)$$

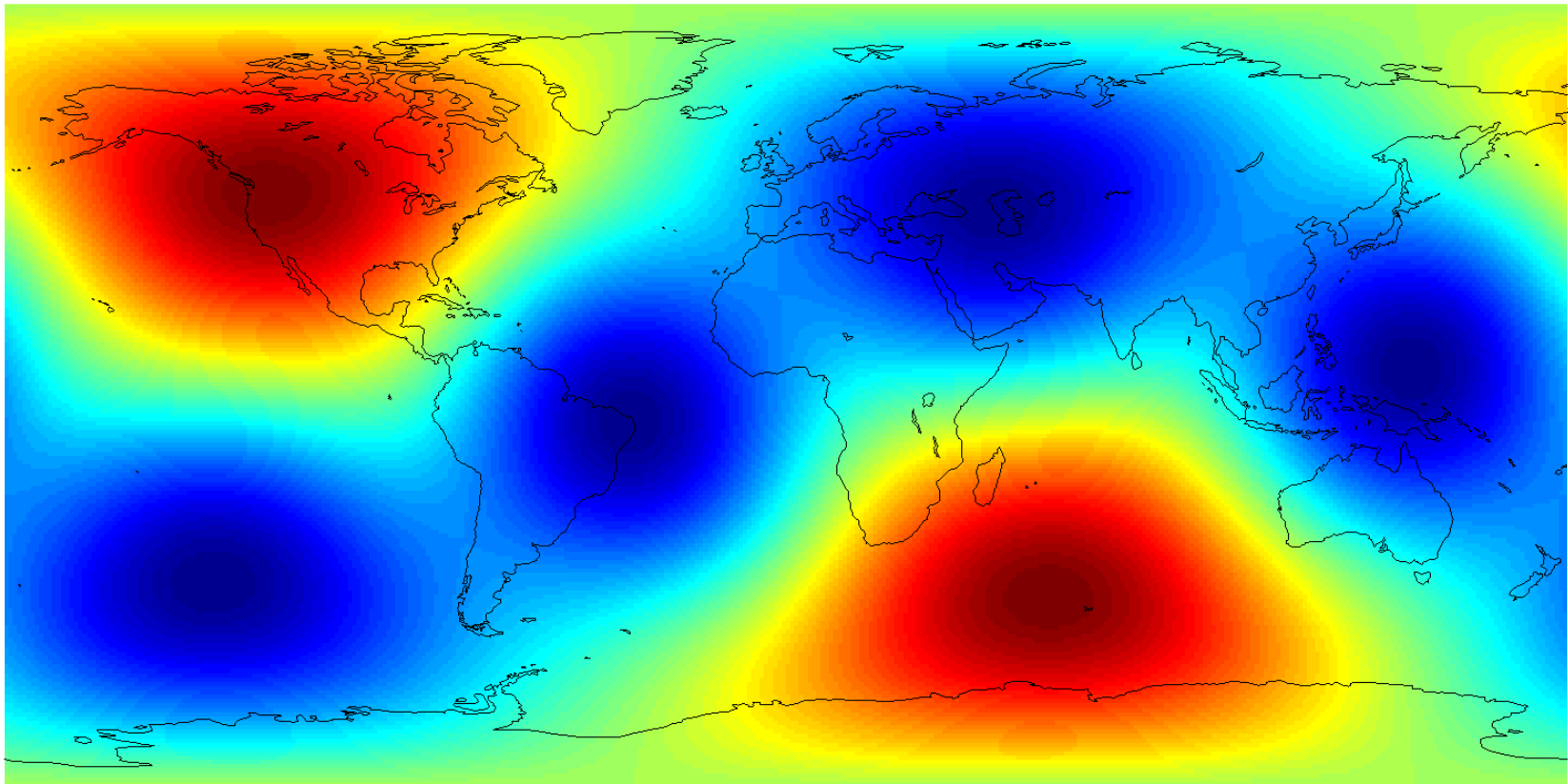
$$F_x = \frac{1}{2} (1 + \cos^2(\Theta)) \cos(2\Phi) \sin(2\Psi) - \cos(\Theta) \sin(2\Phi) \cos(2\Psi)$$

Detector
Polarization



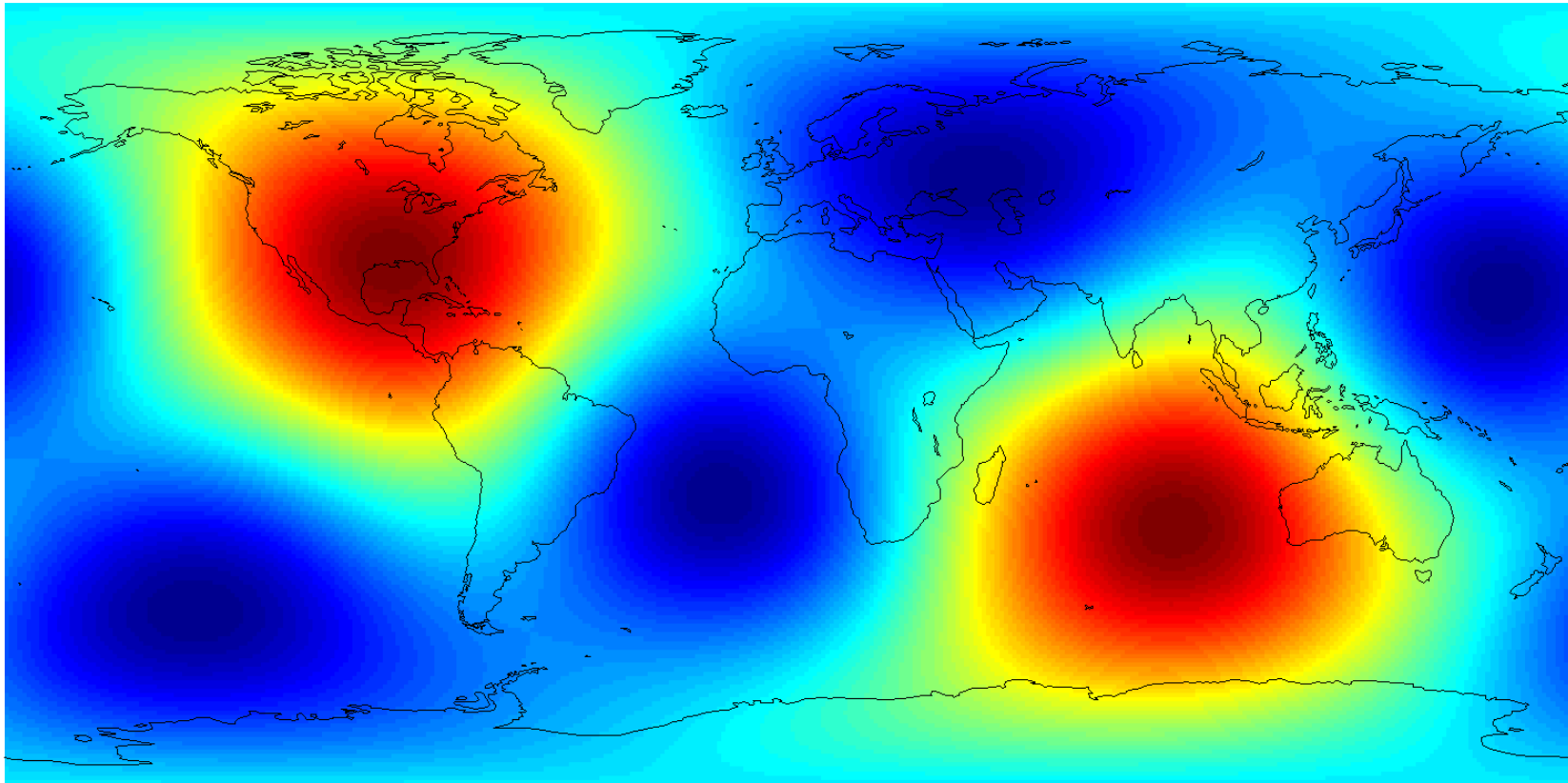
Beam pattern : Hanford

$$F = F_+^2 + F_x^2$$



Beam pattern : Livingston

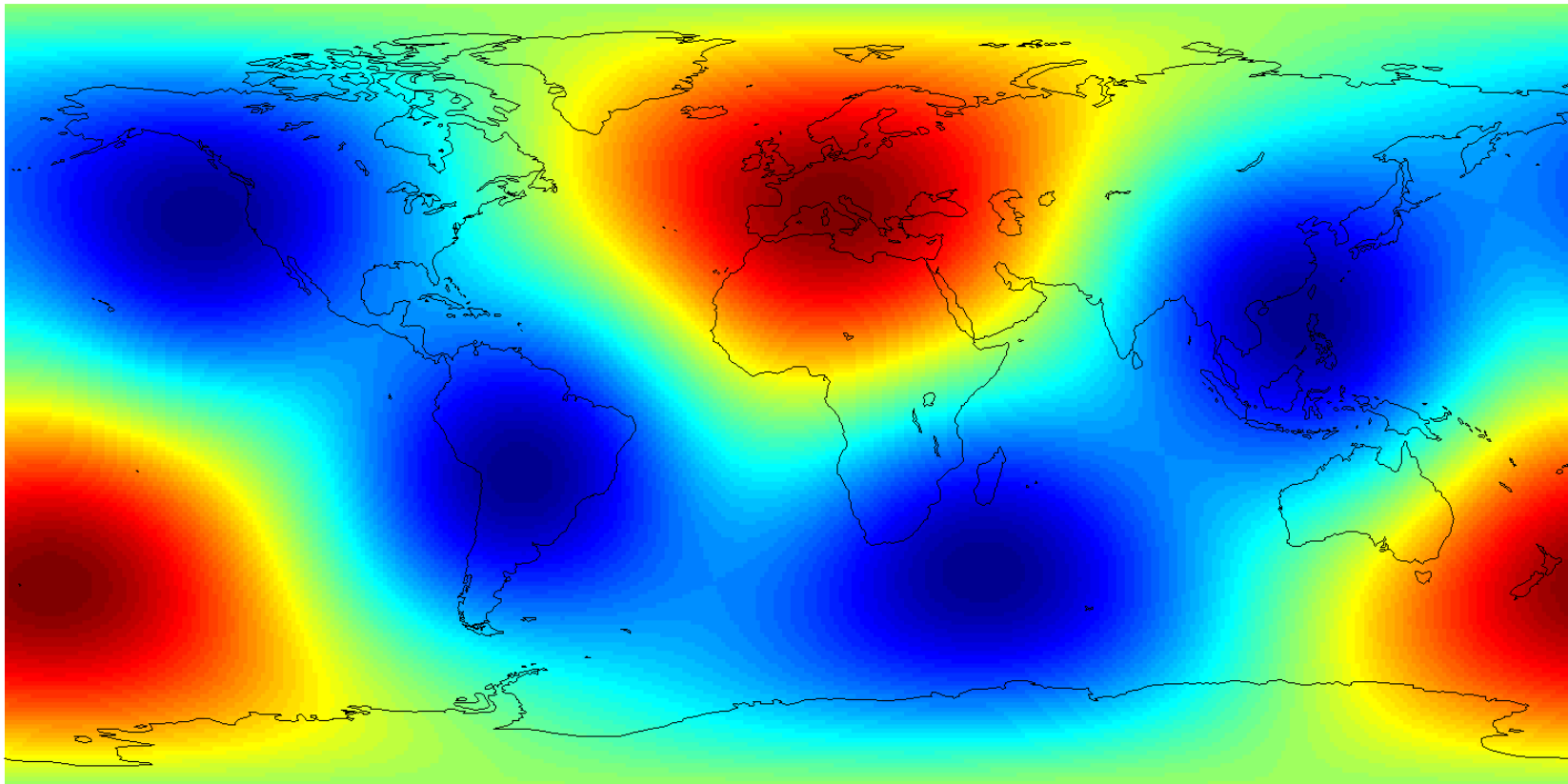
$$F = F_+^2 + F_x^2$$



LIGO detectors are almost co-aligned

Beam pattern : Virgo

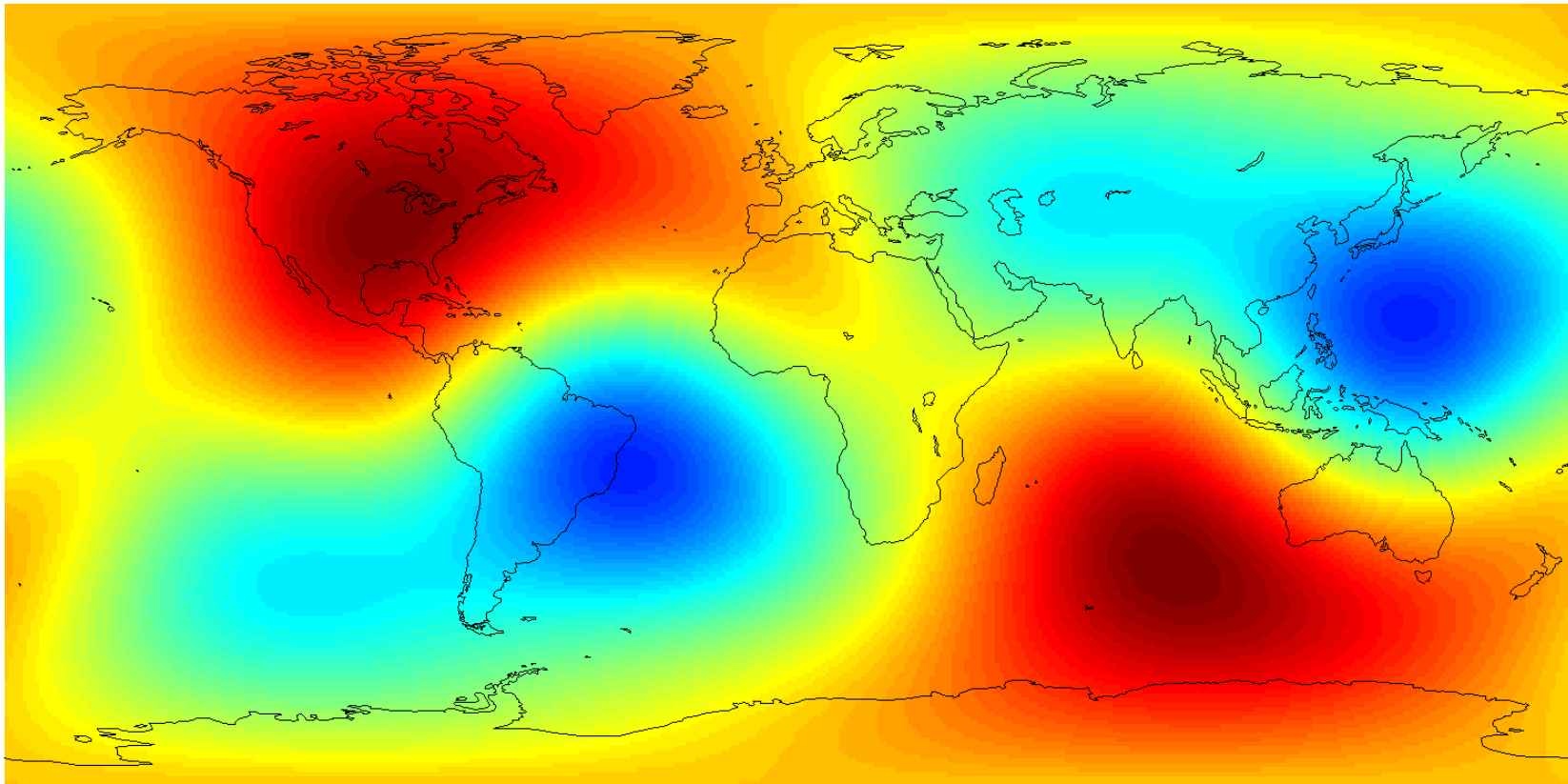
$$F = F_+^2 + F_x^2$$



Virgo detector is quite misaligned

On the LIGO-Virgo network

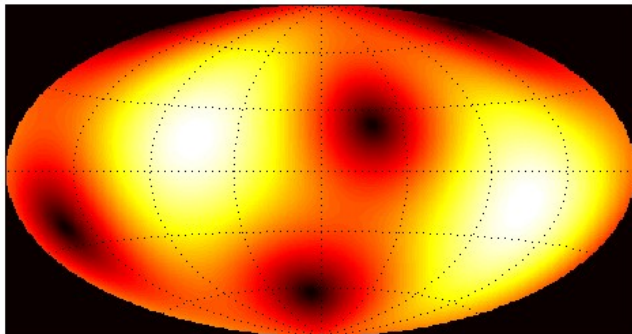
$$F = \sum_i F_{+,i}^2 + F_{x,i}^2$$



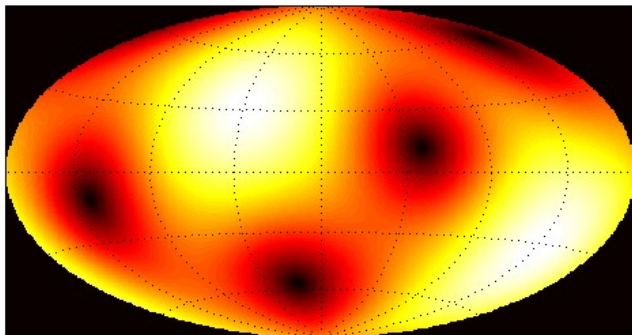
Using sky coordinates

Galactic coordinates

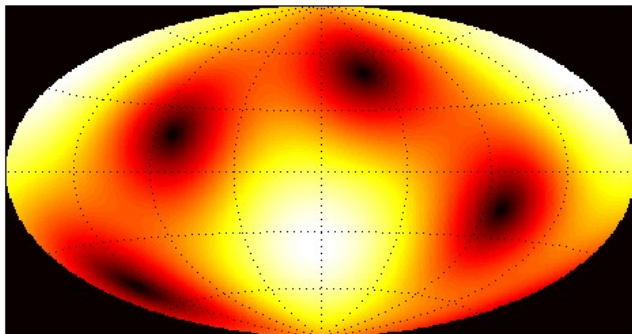
- Earth rotation effect
- Thanks to the network we can cover almost the full sky



H1



L1



V1

$$\begin{pmatrix} F_+ \\ F_x \end{pmatrix} = \begin{pmatrix} \cos(2\psi) & \sin(2\psi) \\ -\sin(2\psi) & \cos(2\psi) \end{pmatrix} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$a(t) = \frac{-1}{16} \sin(2\gamma)(3 - \cos(2l))(3 - \cos(2\delta)) \cos(2\Omega) - \frac{1}{4} \cos(2\gamma) \sin(l)(3 - \cos(2\delta)) \sin(2\Omega) - \frac{1}{4} \sin(2\gamma) \sin(2l) \sin(2\delta) \cos(\Omega) - \frac{1}{2} \cos(2\gamma) \cos(l) \sin(2\delta) \sin(\Omega) - \frac{3}{4} \sin(2\gamma) \cos^2(l) \cos^2(\delta)$$

$$b(t) = -\cos(2\gamma) \sin(l) \sin(\delta) \cos(2\Omega) + \frac{1}{4} \sin(2\gamma)(3 - \cos(2l)) \sin(\delta) \sin(2\Omega) - \cos(2\gamma) \cos(l) \cos(\delta) \cos(\Omega) + \frac{1}{2} \sin(2\gamma) \sin(2l) \cos(\delta) \sin(\Omega)$$

$$\Omega = \kappa t - (\alpha + L) + T_{Greenwich}(0)$$

Source: α (right ascension), δ (declination), ψ (polarization)

Detector: l (latitude), L (longitude), γ (bisectrix orientation (towards north))

Earth rotation: κ

Coincident analysis

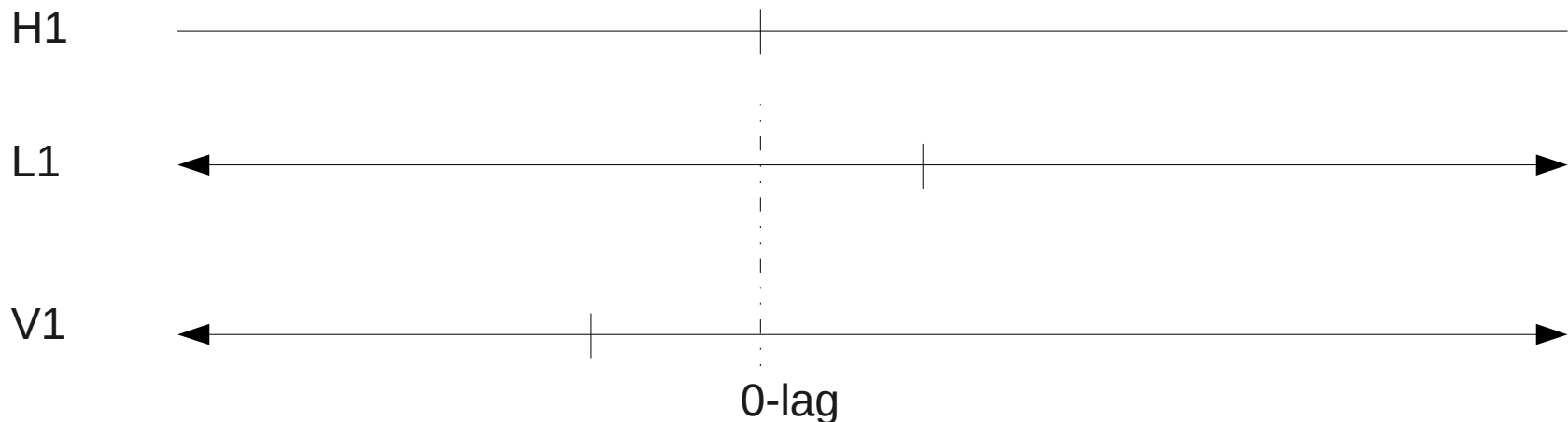
- From each ITF, triggers with:
 - Amplitude
 - Frequency
 - Time
- Try to find coincident events in time:
 - If low trigger rate
ie Event duration $<$ diff time consecutive events
- Adjusting correctly time window

Strategy of analysis

- When use have more than 2 detectors not necessary co-aligned (like LIGO-Virgo)
 - Ask for a triple coincidence
 - Reduce a lot our FA probability and increase our confidence
 - Reduce also your sky coverage to 20 %
 - Ask for an “OR” of double coincidences
 - Increase a lot the sky coverage and your probability to detect
- Next question : how significant is your events distribution ?

Determine significance

- Network analysis give a powerful way to estimate background
- Noise is (almost) uncorrelated
- You can “simulate” a longer time acquisition by time-sliding your data streams (or triggers)
 - You are sure to not have any GW events



Determine significance - 2

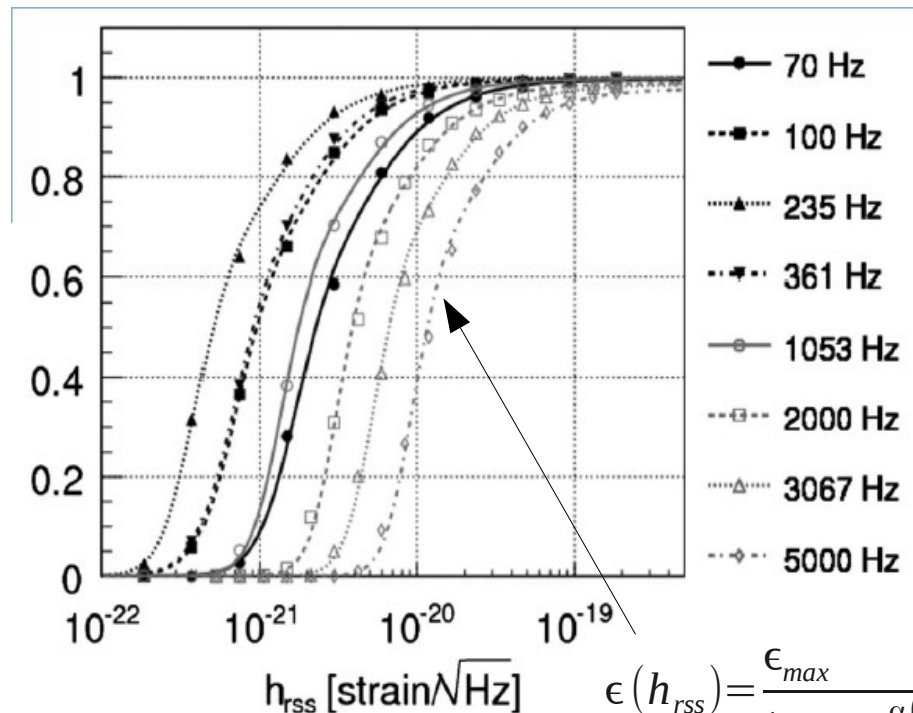
- Do not use “typical” time scale for the time step
 - Suspensions frequencies,
- Do not count 2 times the same configuration
- It is better to not shift too much to avoid long scale coherence:
 - Day time, sea activity, ..
- we must use time shift larger than true signal
- How many time shift ?
 - If you want to check a 5σ level, you need to reach a probability of $5.7 \cdot 10^{-7}$
 - For one year of living time, you need to accumulate 1750000 years

Determine significance - 3

- As we have no GW events in the time-shifted data set, the events must follow a Poisson distribution
- Take properly into account the change of live time for each time slide
- Put your threshold on the false alarm probability:
 - Lower threshold increase efficiency for detection
 - But also increase your number of background triggers

Software injections

- If you have a null result in your coincidence
 - Put an upper limit
- You have detection and you want to give results on the rate
- You need to determine the efficiency of the analysis
- Inject coherently software injections
 - Time delays
 - Amplitudes
 - Take into account astrophysics prior (if possible)



$$\epsilon(h_{rss}) = \frac{\epsilon_{max}}{1 + r_{mid}^{\alpha(1 + \beta \tanh(r_{mid}))}}$$

with $r_{mid} = h_{rss} / h_{rss}^{mid}$

Open the box

- Now you can analyze the 0-lag configuration
- If you have event(s) above your threshold :
 - Check your event(s) !
 - Extensive follow-up !
 - Lot of excitation !
- If you have nothing or kill all events
 - Put upper limits

Upper limit

- The main question is : how do you define your threshold when performing efficiency :

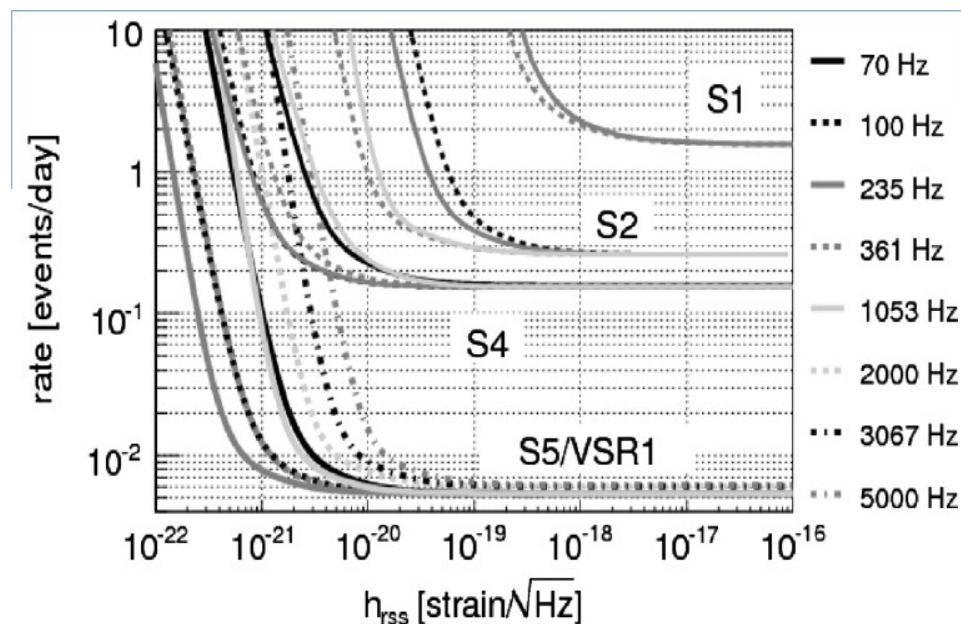
- Loudest event
- False alarm rate

- Then you can compute the upper-limit on event rate with

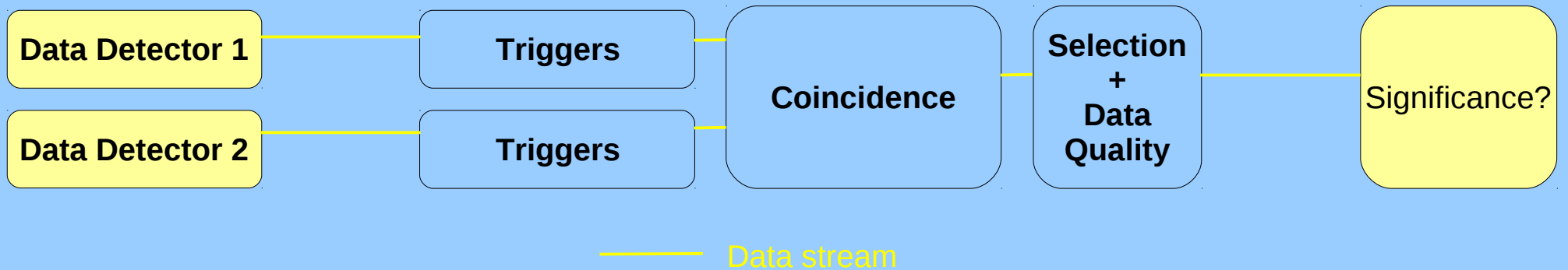
$$\mathcal{R}_{90\%}(h_{rss}) = \frac{-\ln(1-0.9)}{T \epsilon(h_{rss})}$$

- You can also try to translate in distance vs energy

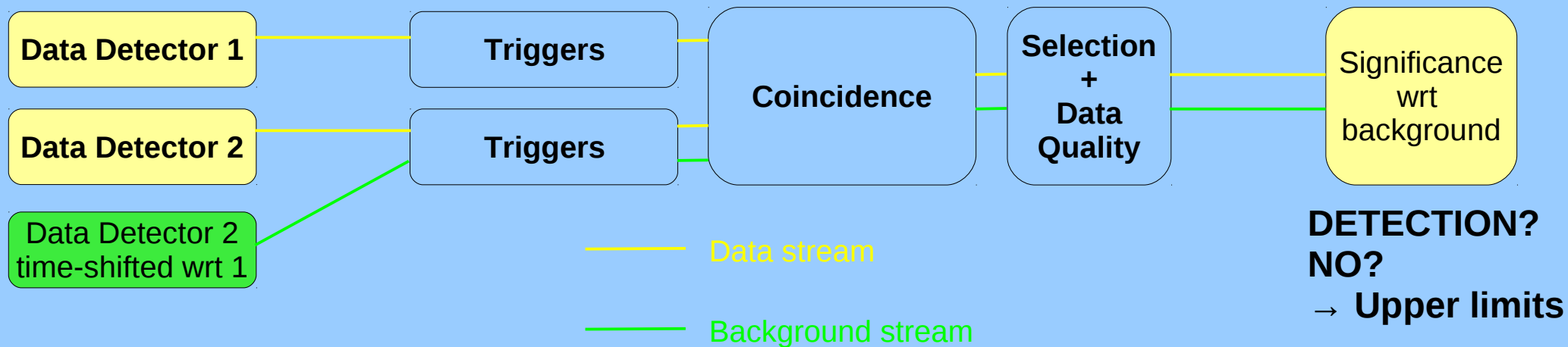
$$\text{for a SG: } E_{GW} \simeq \frac{D^2 c^3}{4G} (2\pi f_0)^2 h_{rss}^2$$



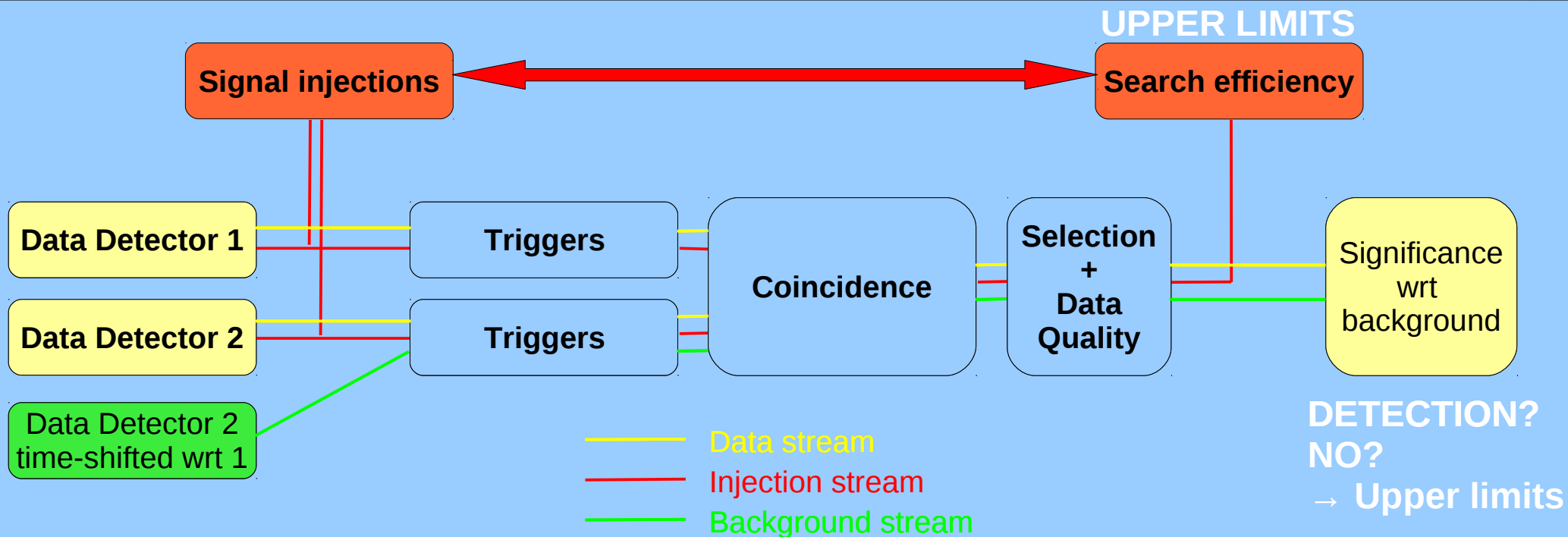
Recap on coincident analysis



Recap on coincident analysis



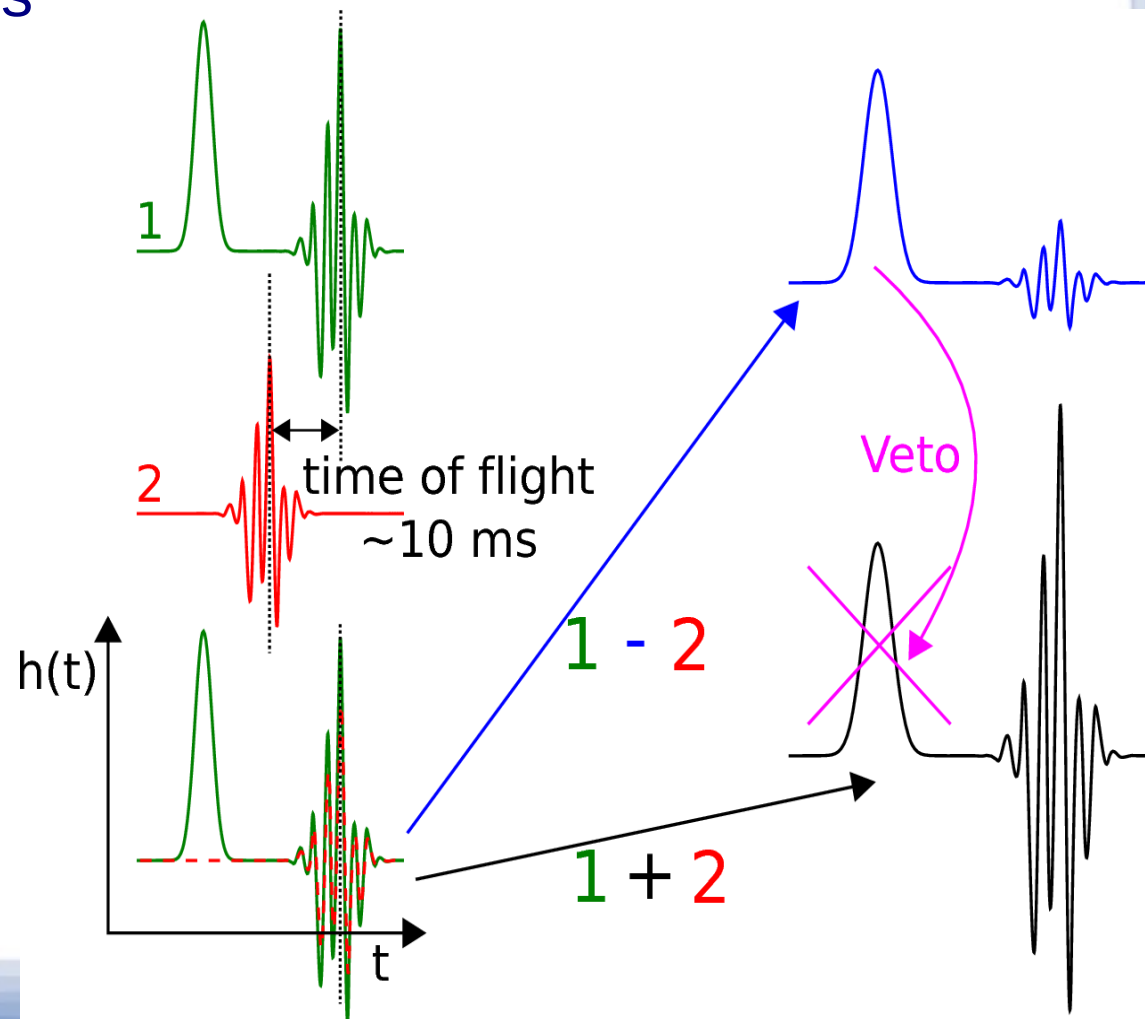
Recap on coincident analysis



Various coincidence schemes: union of configurations
Increasing the number of coincidences enables be more selective (but less efficient)

Coherent analysis

- Idea : sum the data streams to increase the SNR
- For each position on the sky :
 - Compute time delays
 - Shift the streams
 - Take into account beam pattern
 - Take into account sensitivities

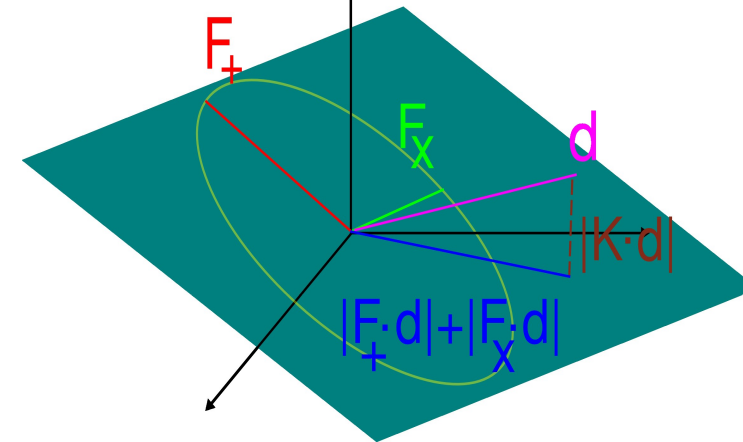


Coherent formalism

- Based on a null stream

$$\begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix} = \begin{bmatrix} F_i^+ / \sigma_i \\ \vdots \\ F_N^+ / \sigma_N \end{bmatrix} h_+ + \begin{bmatrix} F_i^x / \sigma_i \\ \vdots \\ F_N^x / \sigma_N \end{bmatrix} h_x + \begin{bmatrix} n_1 \\ \vdots \\ n_N \end{bmatrix} \rightarrow \mathbf{d} = \mathbf{F}_+^w h_+ + \mathbf{F}_x^w h_x + \mathbf{n}$$

Vector of whitened data time shifted at a given time/frequency for a given sky position



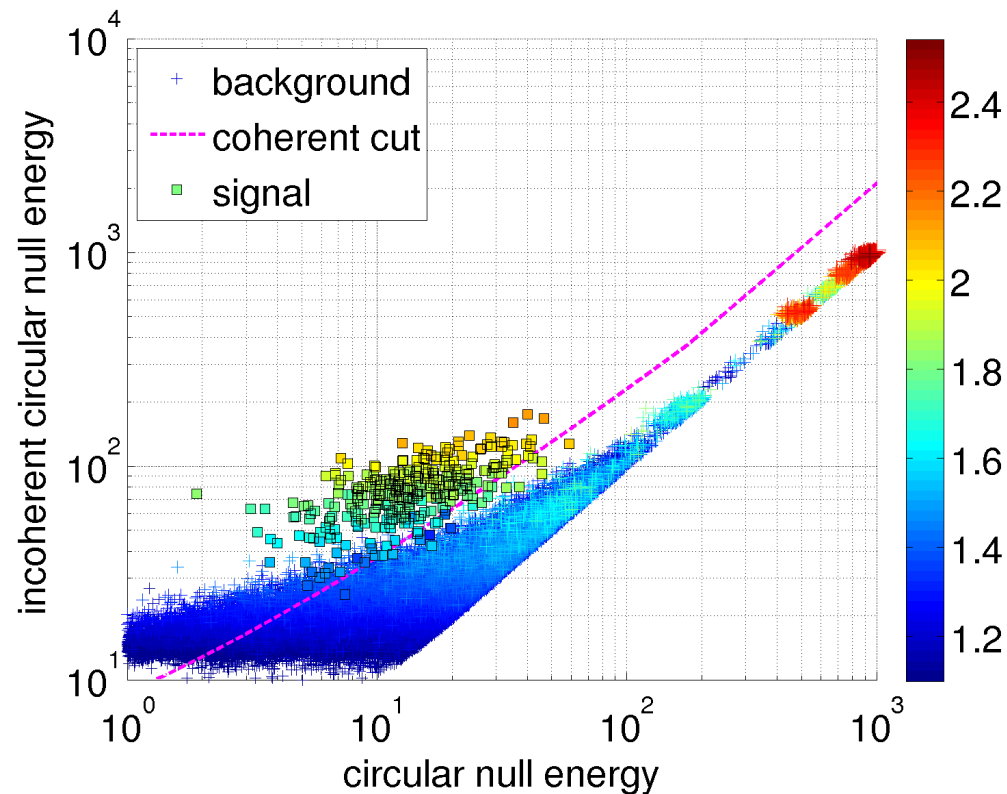
$$E_{tot} = |\mathbf{d}|^2 \quad \text{Total energy}$$

$$E_{null} = E_{tot} - |\mathbf{F}_+^w \mathbf{d}|^2 - |\mathbf{F}_x^w \mathbf{d}|^2 = |\mathbf{K} \cdot \mathbf{d}|^2 \quad \text{Null energy ie noise only}$$

$$E_{inc} = \sum_{\alpha=1}^N |K_{\alpha} d_{\alpha}|^2 \quad \text{Incoherent energy ie contribution from each detector}$$

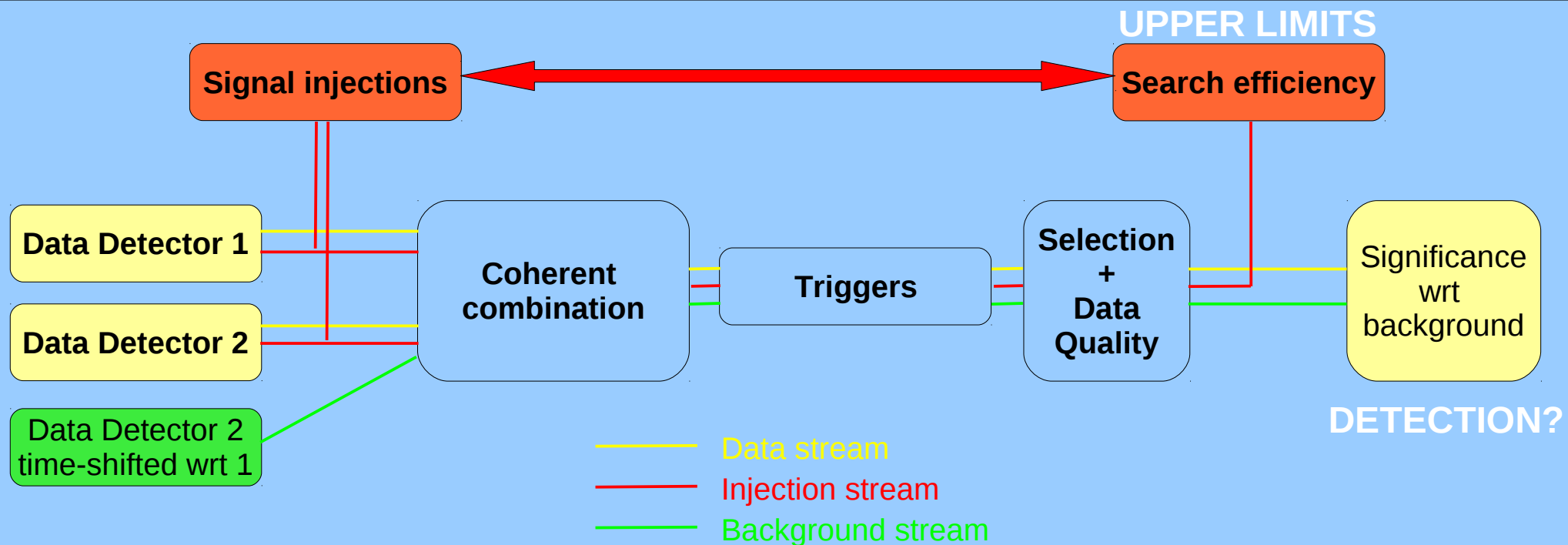
Glitches removal

- Compare null and incoherent energy:
 - GW is cancelled in E_{null} $\rightarrow E_{\text{inc}}/E_{\text{null}}$ is large
 - No cancellation for glitches $\rightarrow E_{\text{inc}}/E_{\text{null}} \sim 1$



Example on GRB analysis

Schematic view for coherent analysis

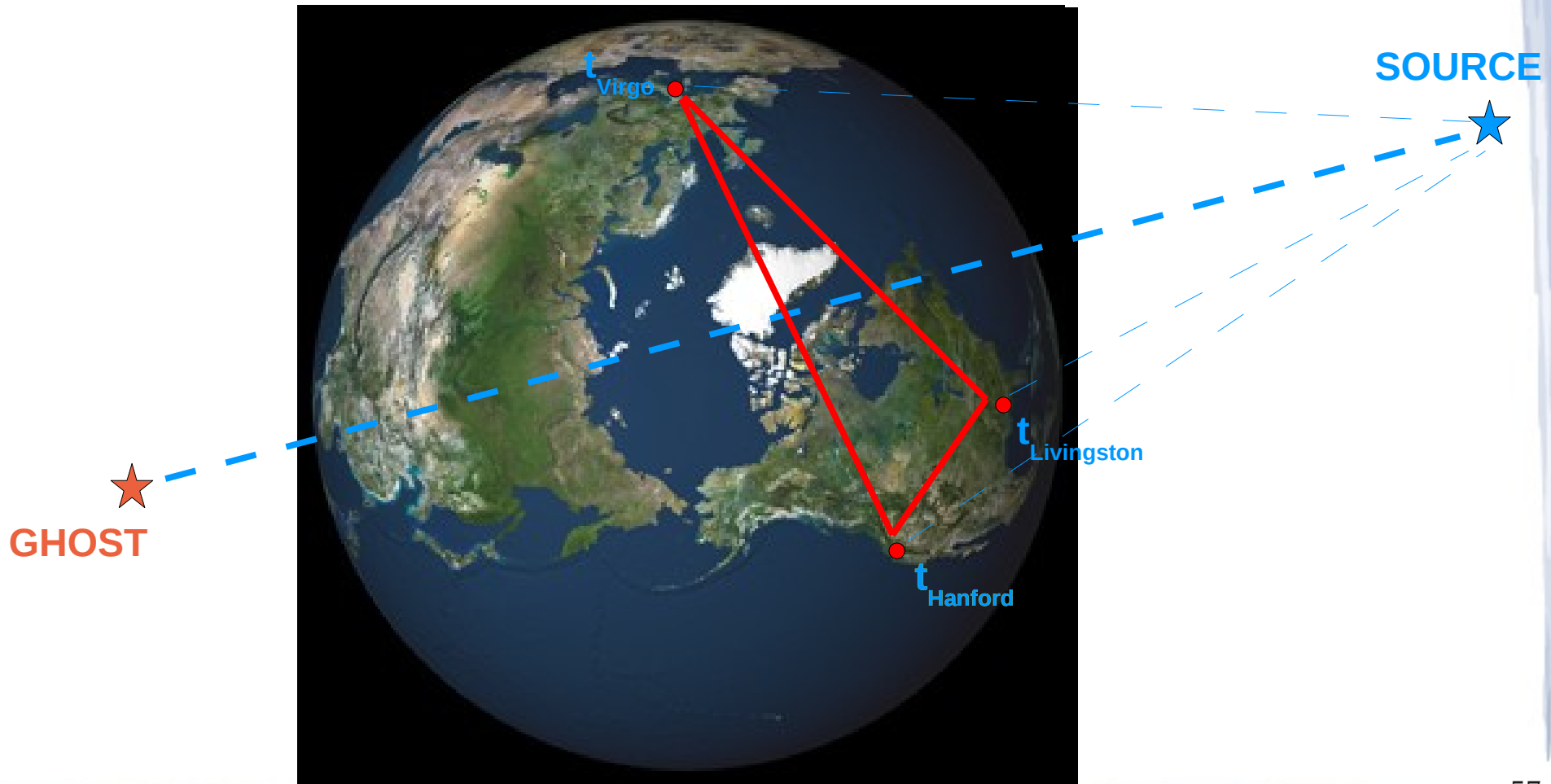


The data of multiple detectors can be combined coherently
Sky positions are scanned to take into account the time of arrival
and the antenna pattern of each detectors

Coincident vs coherent

- Coincident analysis:
 - Faster : one pass is enough
 - More robust to possible errors
- Coherent analysis :
 - Sensitive to weaker signal
 - Scan all sky positions – a null result in one ITF could be used for sky reconstruction
- Or make a coincident analysis and a coherent follow-up

Position Reconstruction

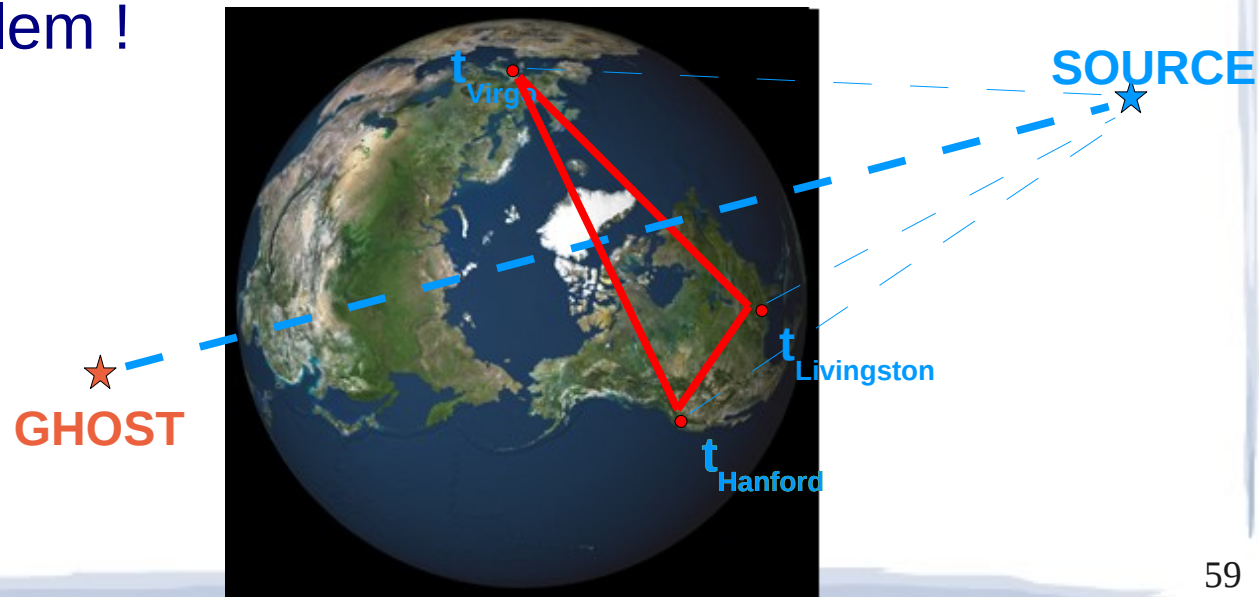


How to reconstruct a position

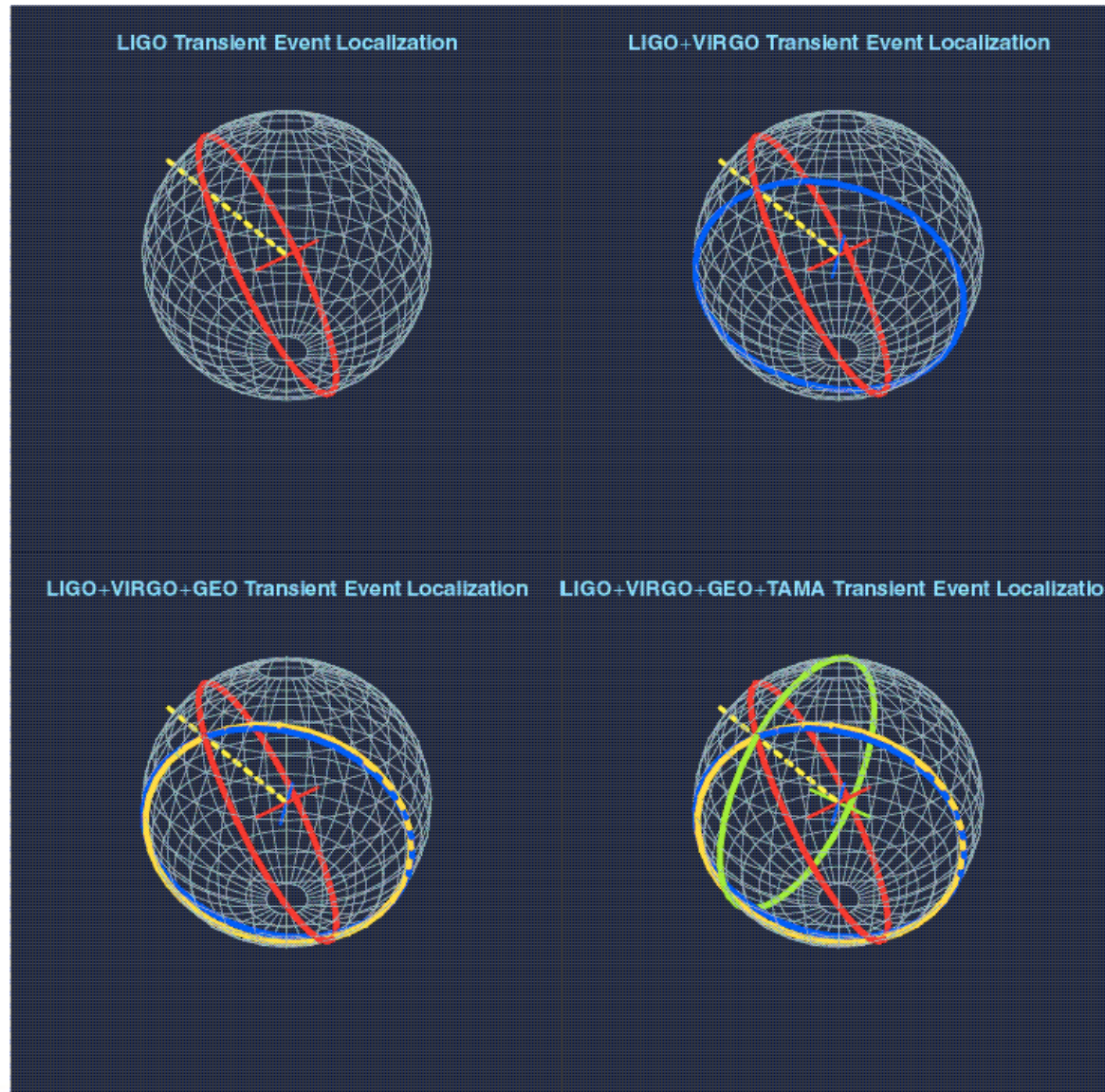
- One instrument and no external observation : no way
- 2 sites (with data) :
 - One time delay measurement
 - All directions giving the same delay are equiprobable
 - Circle on the sky map

How to reconstruct a position

- 3 sites:
 - If all detector observed a signal -> triangulation
 - But two possible directions
 - If only 2 observed a signal -> you can put constraints on the circle
- 4 sites (well separated):
 - No more problem !



Circles on the sky



Using external triggers

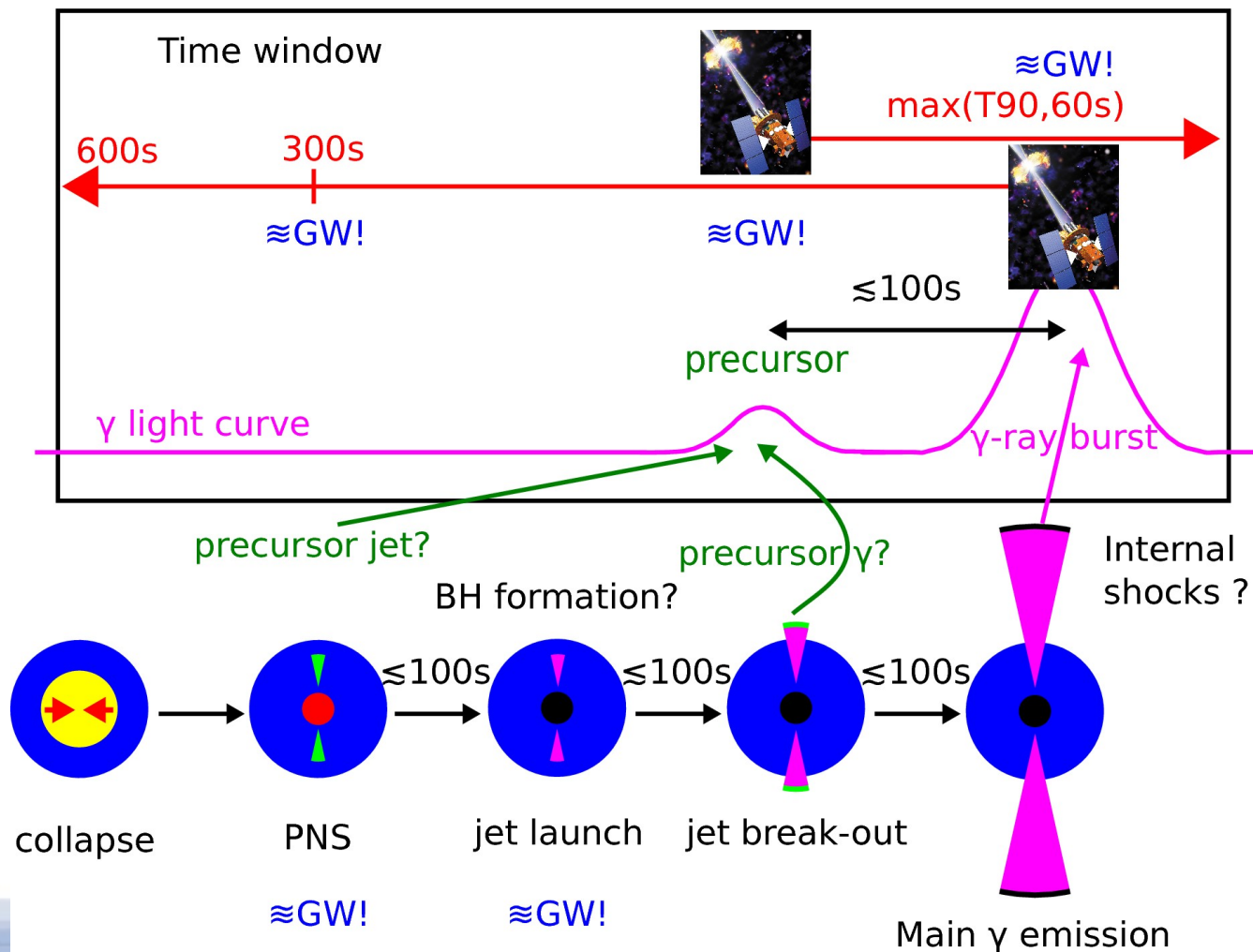
- EM observatories release alerts/information on interesting events like for GRB (GCN)
- Get time and positions
 - Position : simplify for coherent analysis
 - Time : reduced background

can gained up to a factor 2 in sensitivity

- You can also used other messengers like neutrinos from ANTARES/Ice Cube

Use a correct time window

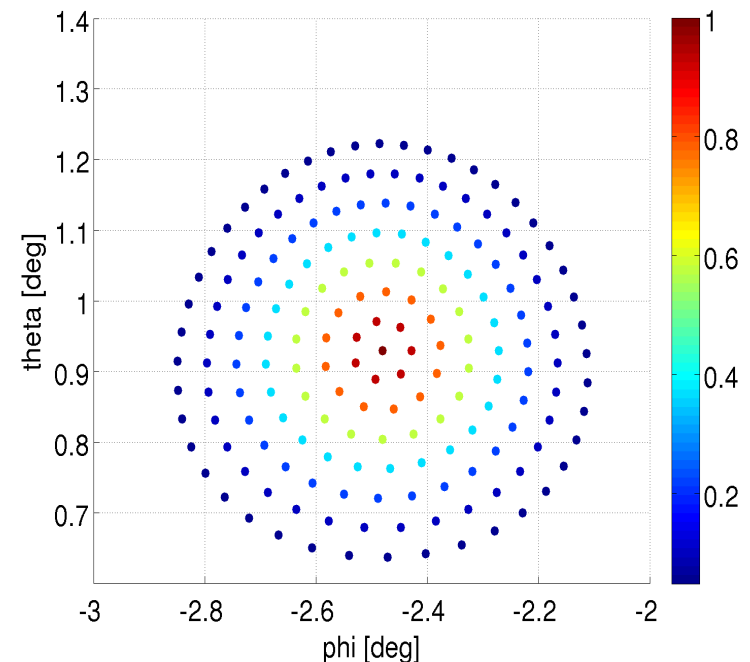
- With an external trigger, you need to define the time window on which searching for the GW
 - Need to take into account astrophysical scenario



Take into account for sky error position

- Limited resolution on the external observatories
 - From few arcminutes (BAT - Swift) to few degrees (GBM-Fermi)
 - Can change the coherent calculation
 - Tests different sky positions

Example using
GBM-Fermi trigger
Color scale is the probability



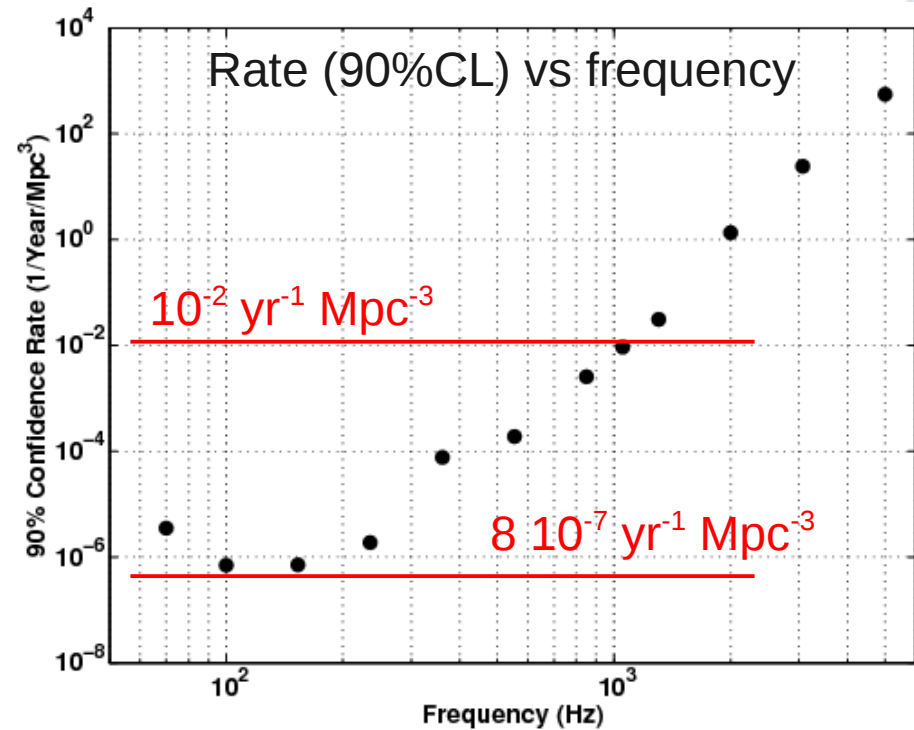
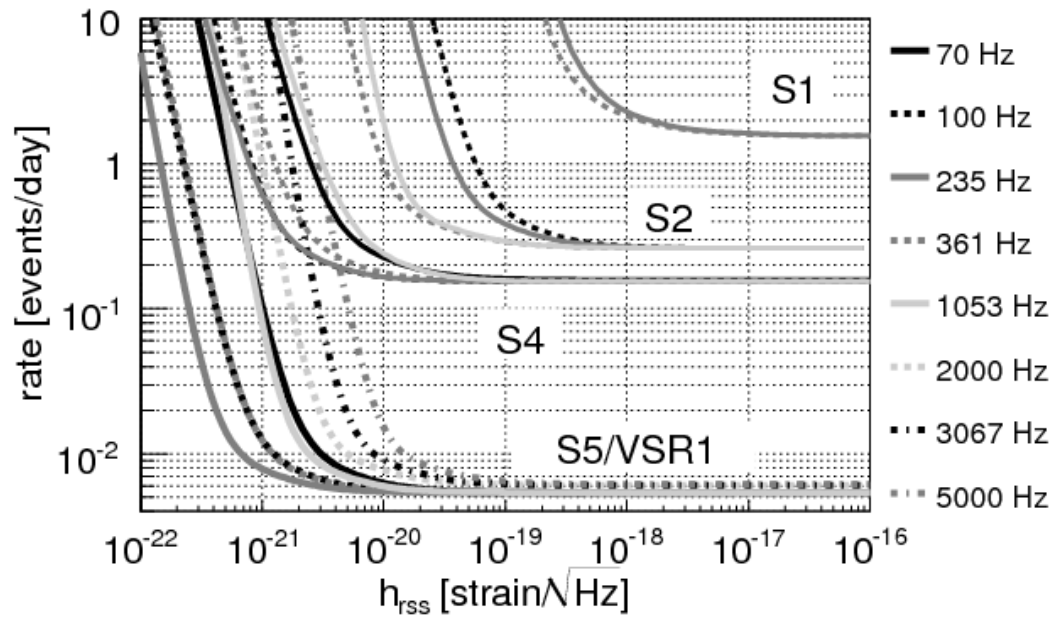
What else?

- $h(t)$ signal is done after calibration and reconstruction of the transfer function of our instrument:
 - From dark fringe photodiode (V) to a dL/L
- 2 possible type of errors:
 - Amplitude error : 10 to 20 %
 - Phase offset -> timing error : $\sim 10\mu\text{s}$
 - Differences between sites
- All pipelines have a resolution also in timing : few ms

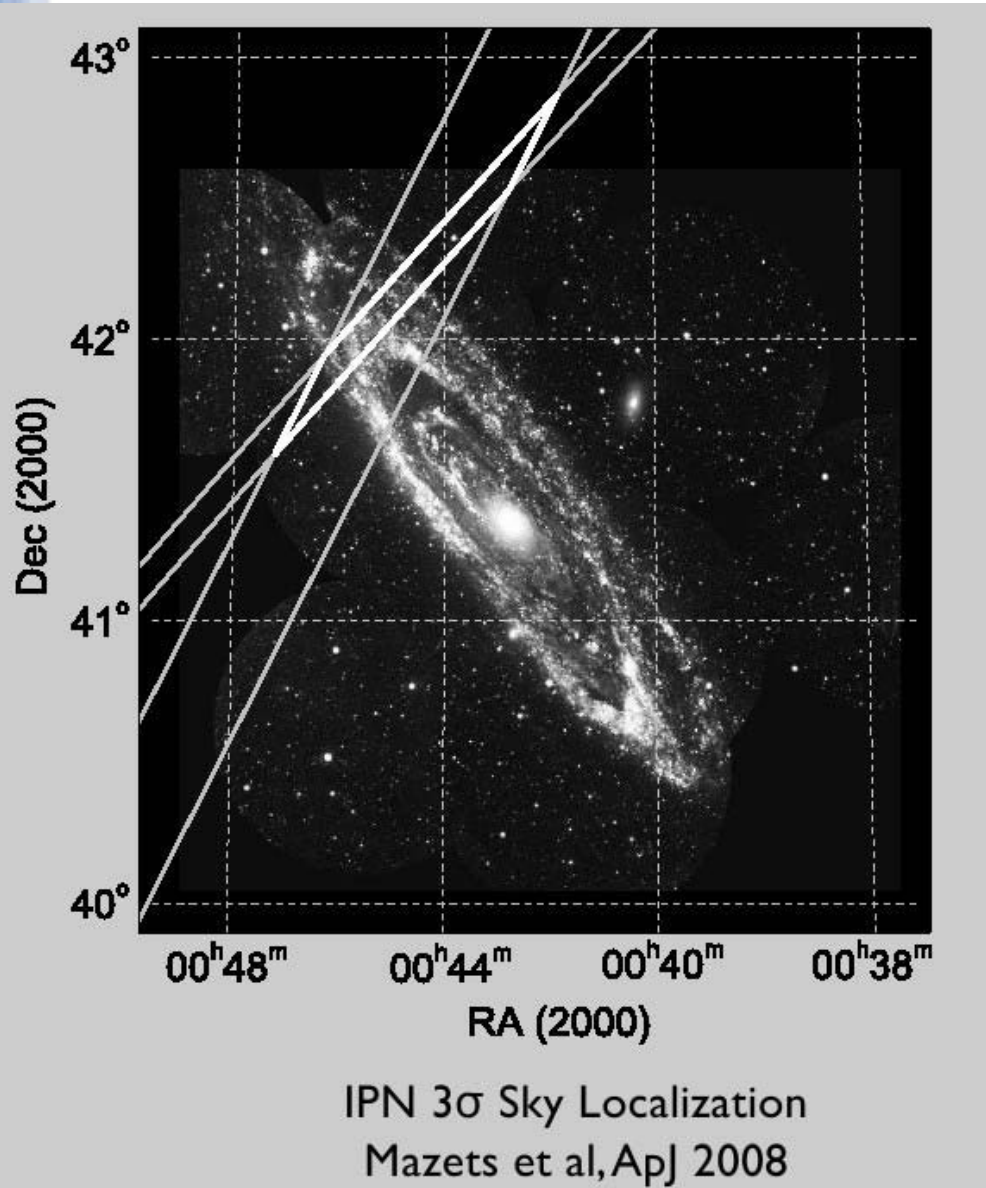
Some recent results

All-sky searches

- Not yet a detection
- Put limits on events rate
 - $50 < f < 2048$ Hz \rightarrow rate < 2 events/year



GRB 070201



- Short GRB detected in direction of Andromeda galaxy
- Both CBC and GRB analysis
- Coalescence scenario in M31 excluded at 99 % CL
- SGR scenario in M31 still possible
- Coalescence in a galaxy behind M31

Some references

- Supernovae and GW : Ott, CQG 26, 063001, 2009
- Sources (a bit old but give hint on h amplitude): Thorne, in 300 years of gravitation, Univ. of Chicago Press
- A.-C. Clapson, PhD thesis Univ. Paris-Sud, 2006
- M. Was, PhD thesis, Univ. Paris-Sud, to be published
- LIGO/S1 paper :B. Abbott et al., PRD 69, 102001 (2004)
- LIGO/S2 paper :B. Abbott et al., PRD 72, 062001 (2005)
- Virgo/C7 paper : Acarnese et al., CQG 26, 085009 (2009)
- Sky reconstruction : Cavalier et al., PRD 74, 082004 (2006)
- S5/VSR1 all-sky : Abadie et al., PRD 81, 102001 (2010)
- GRB070201 : B. Abbott et al., ApJ 681, 1419 (2008)

What's next ?

- Play with a templates bank – this morning
- Search for GW in a network of detectors – morning and this afternoon

Template

- Compute match between template and signal
- Example under matlab could be find in

`/data/procdata/bufferv29/Bursts/VESF2011matchfiltering/test_prog.m`

- Try to pave 1D
 - One Q
 - Different frequencies
- Try to pave 2D

Network analysis

- VSR1 (summer 2007) data
 - L1 and H1 have been simulated by time shifting V1 triggers
- We will use output of omega triggers
 - SG template bank
 - Triggers already produced for the network (GPS, freq, bandwidth, SNR)
 - DQ category 1 and 2 already applied
- For the coincidences
 - Tool is provided, allow coincidence both in time and frequency
- For efficiency studies, software injections already done

Work to be done

- Estimate coincidence window
 - We will start to use time only – add frequency in a second step
- Studies background using time-slide technique
- Estimate our threshold
- Estimate our efficiency – compare with the ITF sensitivity
- Study the 0-lag

Work to be done - 2

- Directory ($\{\text{Burstsn}\}$)
`/data/procddata/bufferv29/Bursts/VESF2011/network/`
- You need to add : $\{\text{Burstsn}\}/\text{environment.csh}$, in your `.cshrc`
- Code for coincidence : $\{\text{Burstsn}\}/\text{coinc.exe}$, run it without arguments to get help
- Data are under: $\{\text{Burstsn}\}/\text{threshold_5.9}$:
 - `\#_raw` : raw triggers
 - `\#.SG9_*` : raw triggers + software injections Q=9, f=100, 200 and 300Hz
- List of injections : $\{\text{Burstsn}\}/\text{listinjecs_}\#$ (GPS and hrss for each type and ITF)
- List of hrss : $\{\text{Burstsn}\}/\text{hrss_injections.txt}$

Useful definition ?

- Network snr : $\sqrt{\sum_i SNR_i^2}$
- Double network : OR (double coincidences)
- Triple network : AND coincidences
- hrss : $h_{rss} = \sqrt{\int |h_+|^2 + |h_x|^2 dt}$