Data Analysis: Stochastic Background

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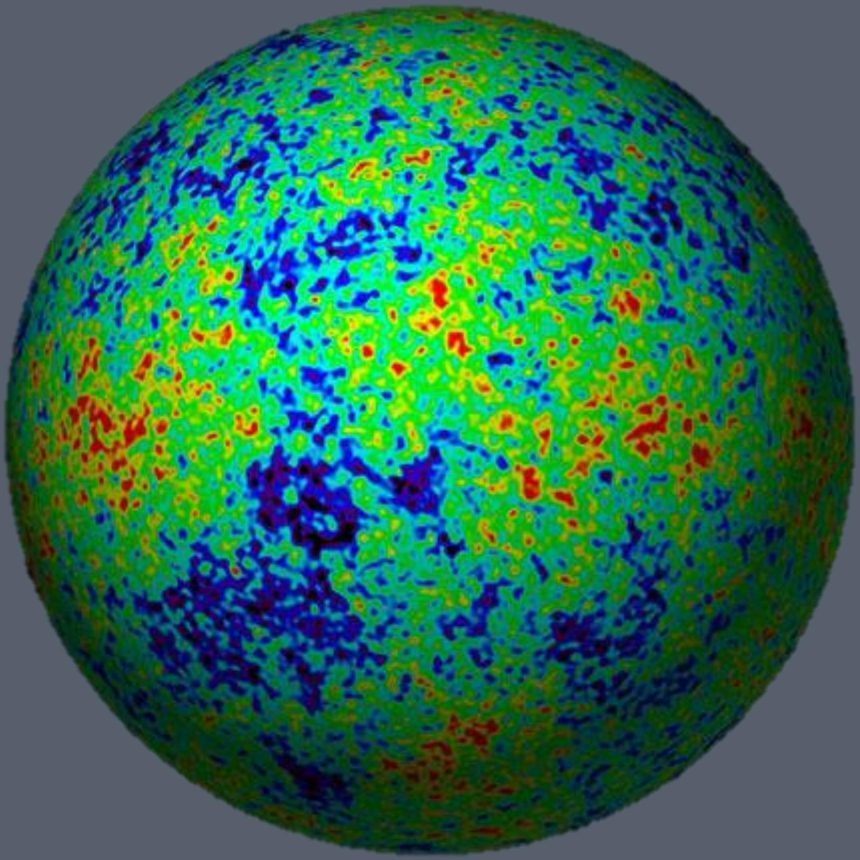


Introduction

What is a stochastic background of gravitational waves

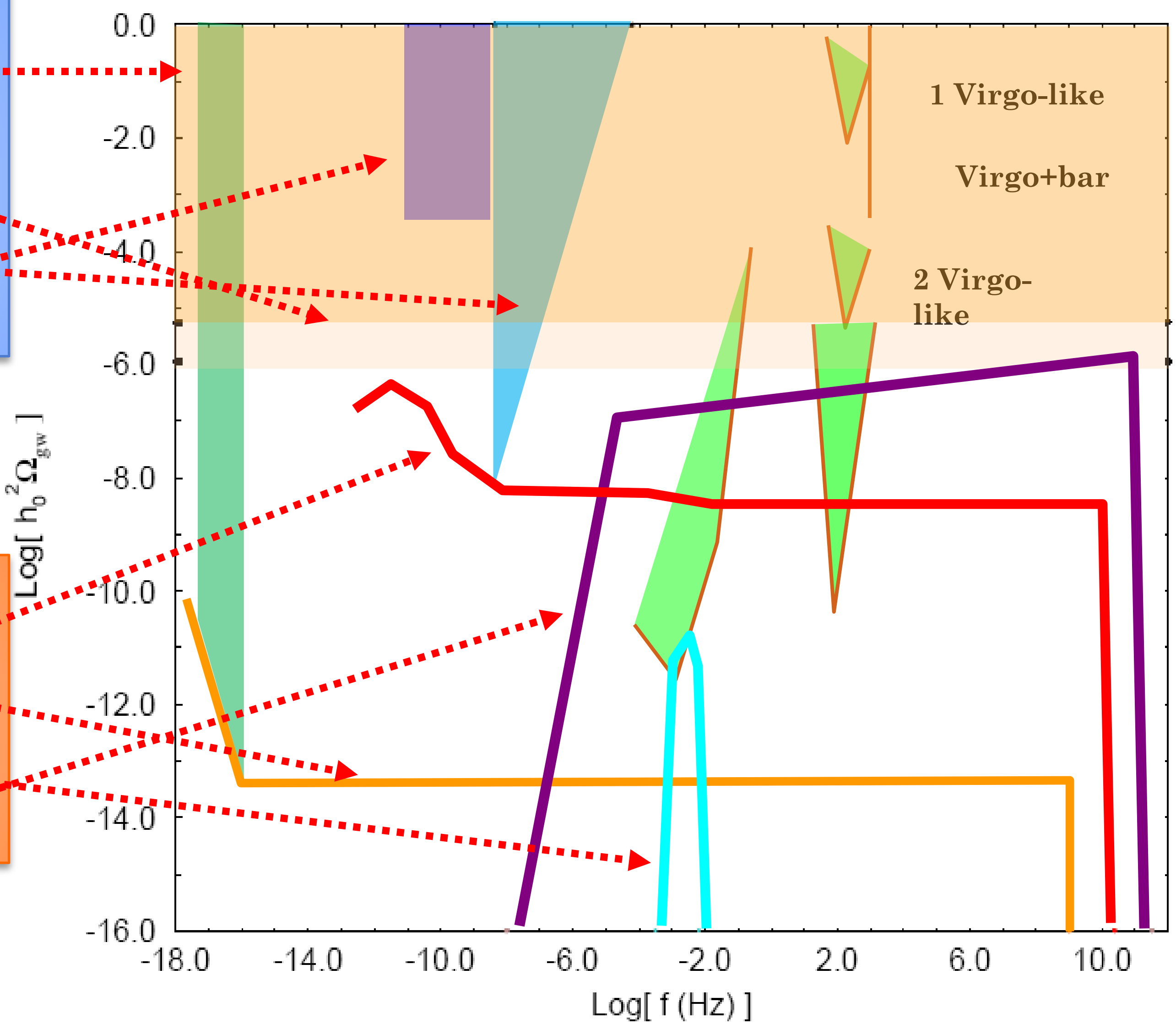
Cosmological Stochastic Background

Upper Bounds:

- Cobe
- Bariogenesis
- Pulsars:
  - millisecond
  - binary

Sources:

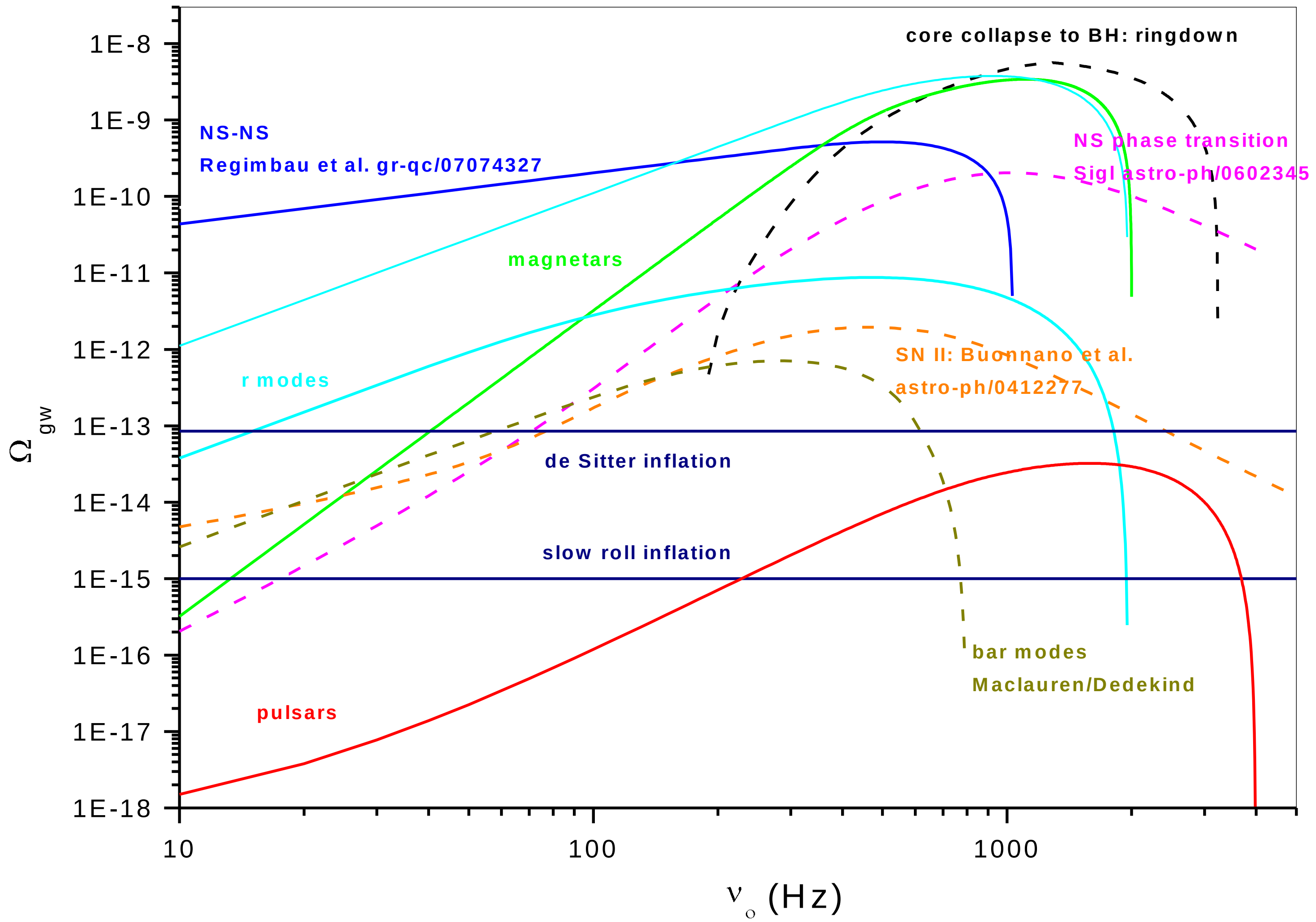
- Inflation
- Cosmic strings
- Phase transitions
- String cosmology



Astrophysical Stochastic Backgrounds of Gravitational Waves

- **Astrophysical Stochastic backgrounds contain informations about**
  - Star formation history
  - Statistical properties of source populations.
- **They may be a noise for cosmological backgrounds**
  - Accurate modelization needed for subtraction
- **Differences with cosmological backgrounds:**
  - anisotropic in the local universe (directed searches)
  - different regimes: Gaussian, **popcorn noise,** Poissonian
  - spectrum characterized by a **maximum** and a cutoff frequency
- **Advanced detectors may be able to put interesting constraints**
  - NS ellipticity, magnetic field, initial period
  - rate of compact binaries
  - ……

Predictions for spectra



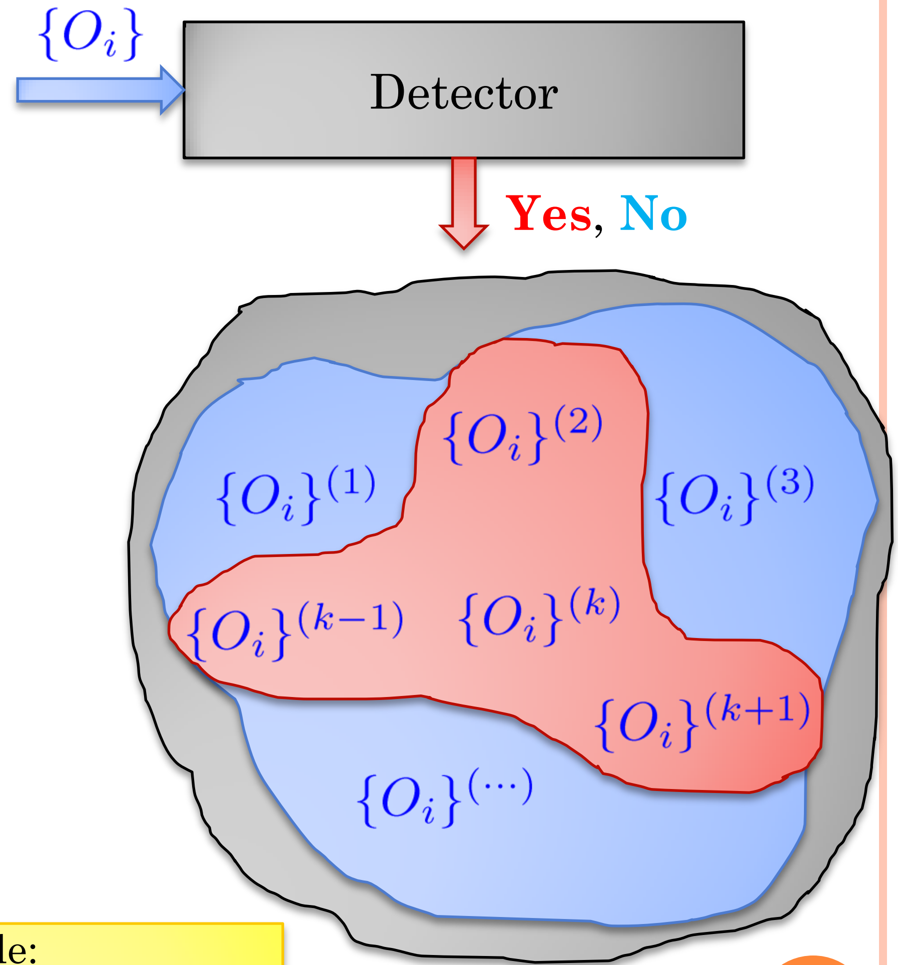
Detection

Frequentist approach


EPREUVE par la BAGUETTE.

The detection problem

- A detector is an arbitrary rule to partition the space of possible experimental results in two parts (yes or no answer to a well defined question)
- We characterize the detector with
  - Detection probability P_D
  - False alarm probability P_{FA}



Given answer \ True answer	Yes	No
No	$1-P_D$	$1-P_{FA}$
Yes	P_D	P_{FA}

Example:
random choice

$$P_D = P_{FA} = 0.5$$

Neyman-Pearson lemma

- Recipe for the optimal detector:
  - Compare the ratio of two conditional probabilities with a threshold
  - The threshold fixes the false alarm probability
  - Larger detection probability for a given false alarm probability

$$\frac{P(X_1 \cdots X_n | \text{yes})}{P(X_1 \cdots X_n | \text{no})} > \lambda$$

X_i: the observed data

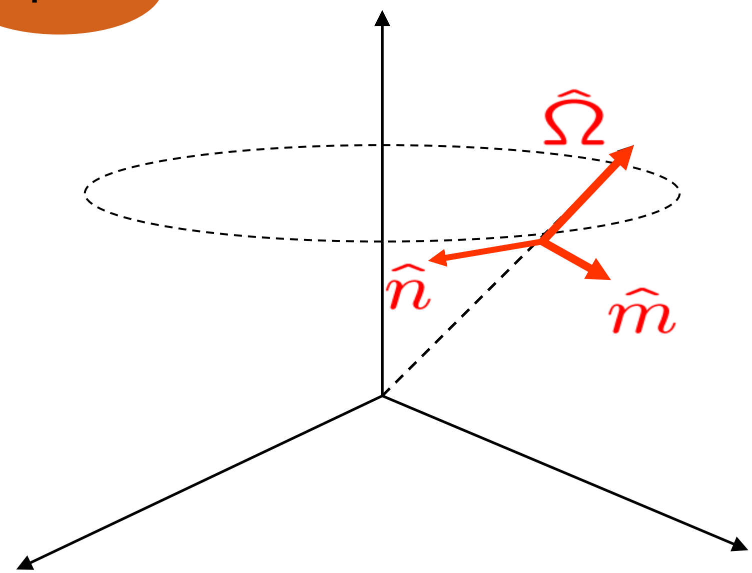
How can we observe a GW stochastic background?

$$h_{ij}(t,\vec{r}) = \sum_{P=+,\times} \int_{S^2} d\hat{\Omega}\, \varepsilon_{ij}^{P}(\hat{\Omega}) \int_{-\infty}^{\infty} df\, \tilde{h}_P(f,\hat{\Omega}) e^{i2\pi f(t-\hat{\Omega}\cdot\vec{r})}$$

Polarizations **Directions** **Frequencies**

$$\varepsilon_{ij}^{+} = m_i m_j - n_i n_j$$

$$\varepsilon_{ij}^{\times} = m_i n_j + n_i m_j$$



$h_{+,\times}(f,\hat{\Omega})$ Stochastic Amplitudes

How we can observe a GW stochastic background?

For a given mode decomposition, we can consider the amplitudes as stochastic variables. What can be said about their statistical properties?

- **Gaussianity** → **Superposition of many independent contributions**
- **Stationarity** → $T_{gw} \ll T_{universe}, T_{obs} \ll T_{universe}$
- **Isotropy** → **"Real" CMB anisotropy:** $\sim 10^{-5}$

If we accept these working hypothesis: **"A stochastic background is completely described by its power spectrum"**

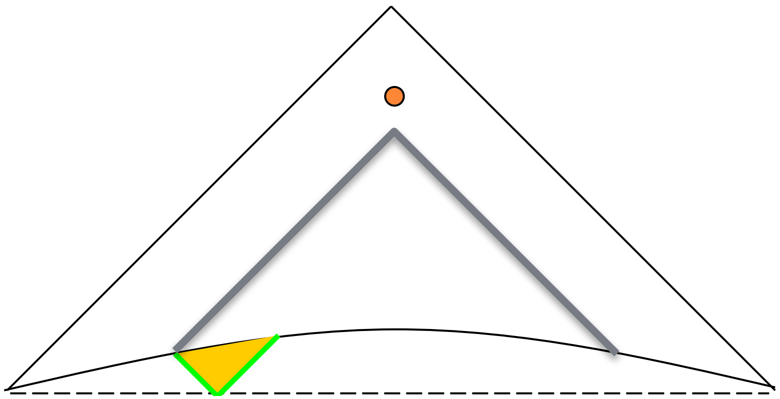
$$< h^{\star}_A(f,\hat{\Omega},\psi) h_B(f',\hat{\Omega}',\psi') >= \delta_{AB}\delta(f-f')\frac{\delta^2(\hat{\Omega},\hat{\Omega}')}{4\pi}\frac{\delta(\psi-\psi')}{2\pi}\frac{1}{2}S_{gw}(f)$$

ARE THESE ACCEPTABLE ASSUMPTIONS?

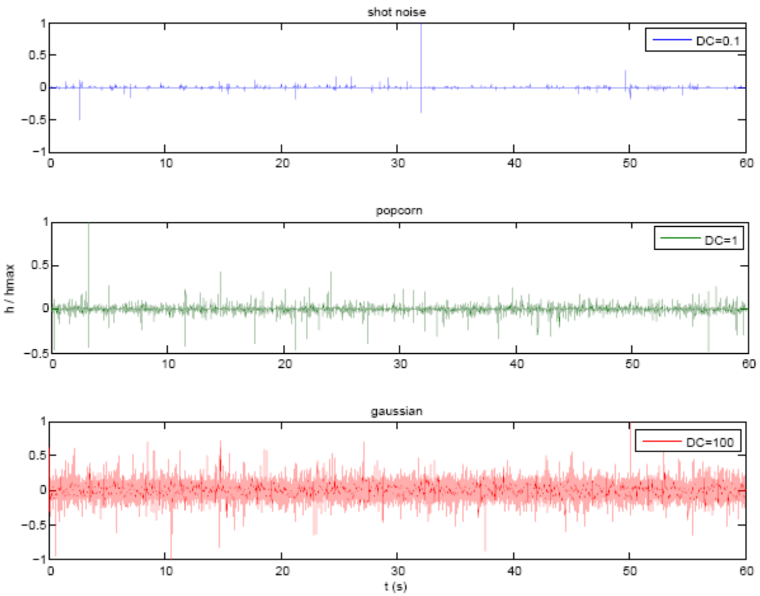
- **Gaussianity:**
  - Cosmological backgrounds: counting independent contributions:

$$1 + Z = \sqrt{\frac{t_0}{t_{gw}}} = \sqrt{\frac{10^{17}}{10^{-22}}}$$

$$N = \frac{A_b}{A_s} = \frac{(\eta_0 - \eta_{gw})^2}{\eta_{gw}^2} = Z^2 \simeq 10^{39}$$



  - Astrophysical backgrounds:
    - Gaussian
    - “popcorn noise”
    - Poisson (resolved sources)



ARE THESE ACCEPTABLE ASSUMPTIONS?

- **Stationarity:**
  - Grishchuk: gravitational fluctuations are in a squeezed state
  - Allen: but there are no observable effects

- **Isotropy:**
  - Not mandatory (e.g. Astrophysical Backgrounds)
  - In principle can be measured (with enough SNR….)

Connection between signal power spectrum and energy density

Inserting in the definition of energy density the mode decomposition we obtain:

$$\rho_{gw} = \frac{1}{32\pi G}\left\langle \frac{dh_{ij}}{dt}\frac{dh_{ij}}{dt}\right\rangle$$

$$\rho_{gw} = \frac{4}{32\pi G}\int_0^\infty df\,(2\pi f)^2 S_h(f)$$

It follows:

$$h_0^2\Omega_{gw}(f) = \frac{1}{\rho_c}\frac{d\rho_{gw}}{d\log f} = \frac{4\pi^2 h_0^2}{3H_0^2} f^3 S_h(f)$$

$$H_0 = H(t_0) = h_0 \times 100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

$$0.50 < h_0 < 0.65$$

Gaussian case: spatial correlations

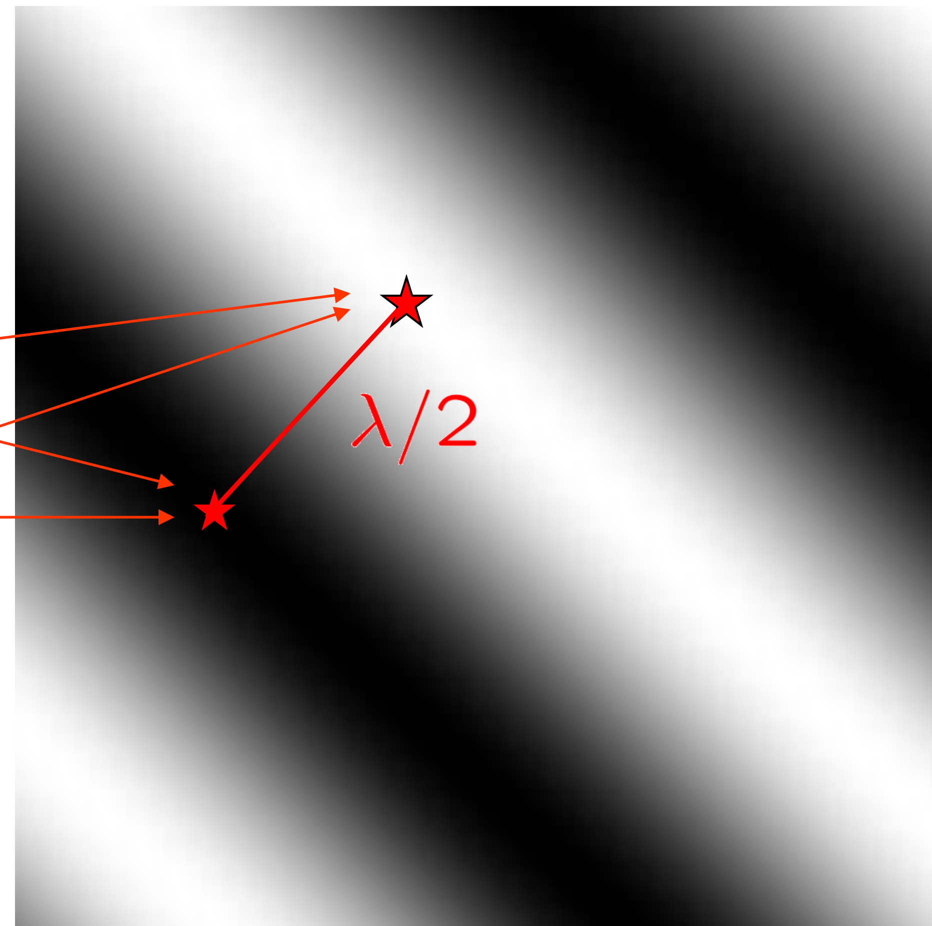
Space correlations can be evaluated analytically. They can be written as a sum over all the modes:

$$\left\langle \tilde{h}^{\star}_{ij}(f,\vec{r})\tilde{h}_{kl}(f',\vec{r}')\right\rangle \propto S_h(f)\delta(f-f')\int d\hat{\Omega}\,\varepsilon^{P}_{ij}(\hat{\Omega})^{\star}\varepsilon^{P}_{kl}(\hat{\Omega})e^{i2\pi f\Delta\vec{r}\cdot\hat{\Omega}}$$

When $|\Delta\vec{r}| \gg \lambda(f)$ the phase factor oscillates strongly on the full solid angle. Correlations go to zero.

Always in phase

Always out of phase



These properties are detector independent. Now we must describe the real measurement.

Coupling to the detector

A detector give us one or more projections of the strain h_{ij} :

$$h(t) = D^{ij} h_{ij}(\vec{r}, t)$$

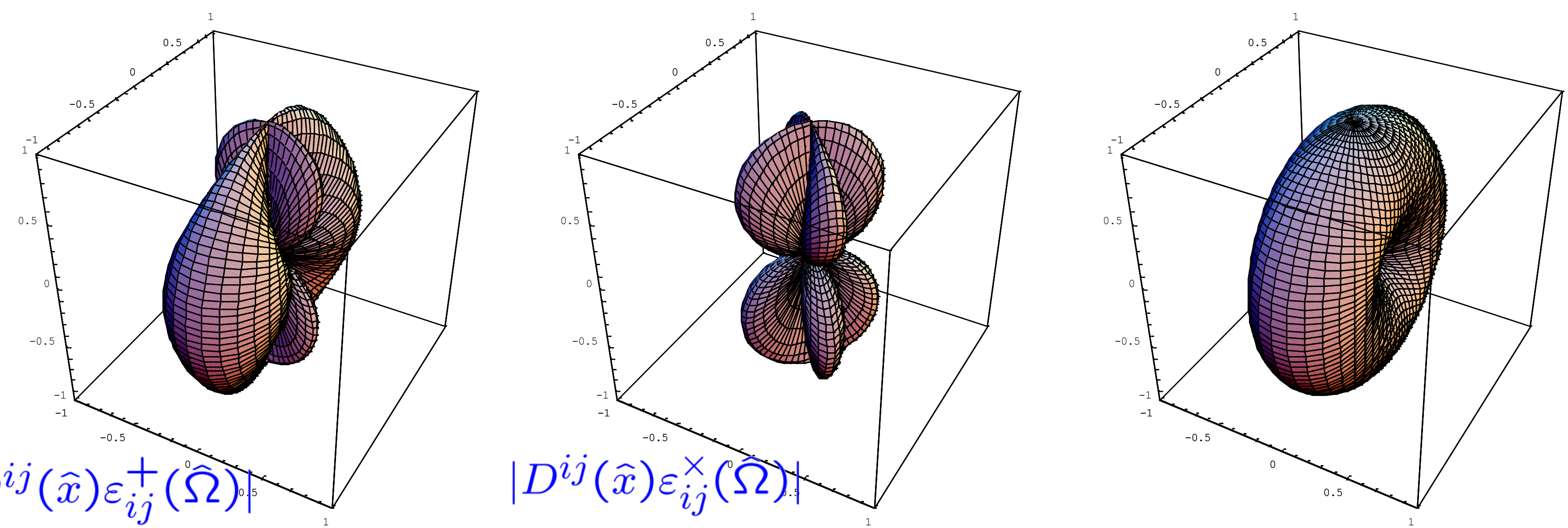
Signal

Detector tensor

Strain at the detector position

Example: for a bar aligned to the $\hat{\ell}$ direction:

$$D^{ij}(\hat{\ell}) = \ell_i \ell_j - \frac{1}{3}\delta_{ij}$$



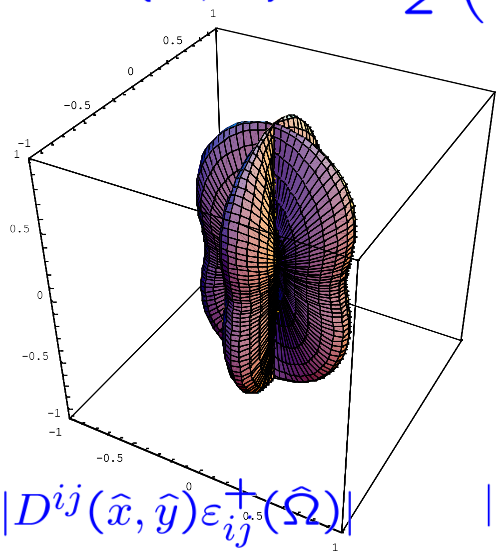
$|D^{ij}(\hat{x})\varepsilon^{+}_{ij}(\hat{\Omega})|$

$|D^{ij}(\hat{x})\varepsilon^{\times}_{ij}(\hat{\Omega})|$

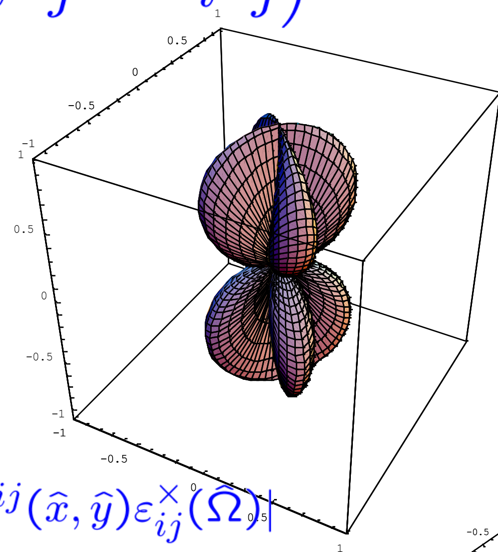
Detector tensor: interferometer

For the differential mode (arms along $\hat{u}, \hat{v}$):

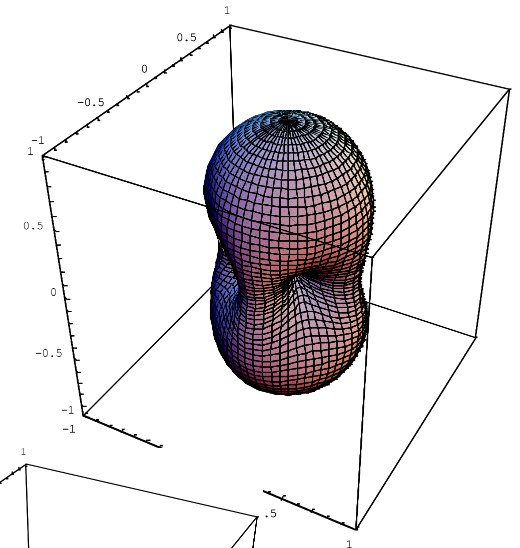
$$D^{ij}(\hat{u},\hat{v}) = \frac{1}{2}\left(u_i u_j - v_i v_j\right)$$



$|D^{ij}(\hat{x},\hat{y})\varepsilon^{+}_{ij}(\hat{\Omega})|$

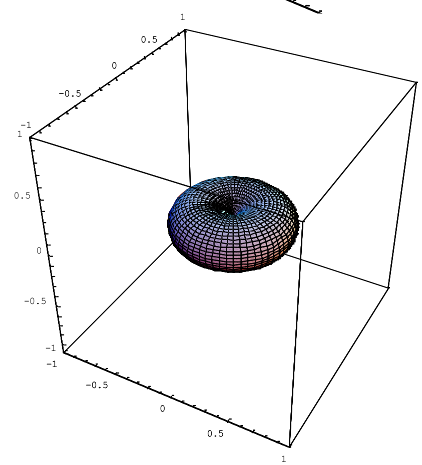


$|D^{ij}(\hat{x},\hat{y})\varepsilon^{\times}_{ij}(\hat{\Omega})|$



Common mode:

$$D^{ij} = \frac{1}{2}\left(u_i u_j + v_i v_j\right) - \frac{1}{3}\delta_{ij}$$



$|D^{ij}(\hat{x},\hat{y})\varepsilon^{+}_{ij}(\hat{\Omega})|$

Overlap reduction function

The signal is a linear combination of the elements of the strain tensor.

- **Gaussian**
- **Stationary, at least if D^{ij} is time independent**

$$\left\langle \tilde{h}^\star_A(f)\tilde{h}_B(f')\right\rangle = \left\langle \left(\tilde{D}^{ij}_A \star \tilde{h}_{ij}\right)^\star (f) \left(\tilde{D}^{kl}_B \star \tilde{h}_{kl}\right)(f')\right\rangle$$

This can be written as

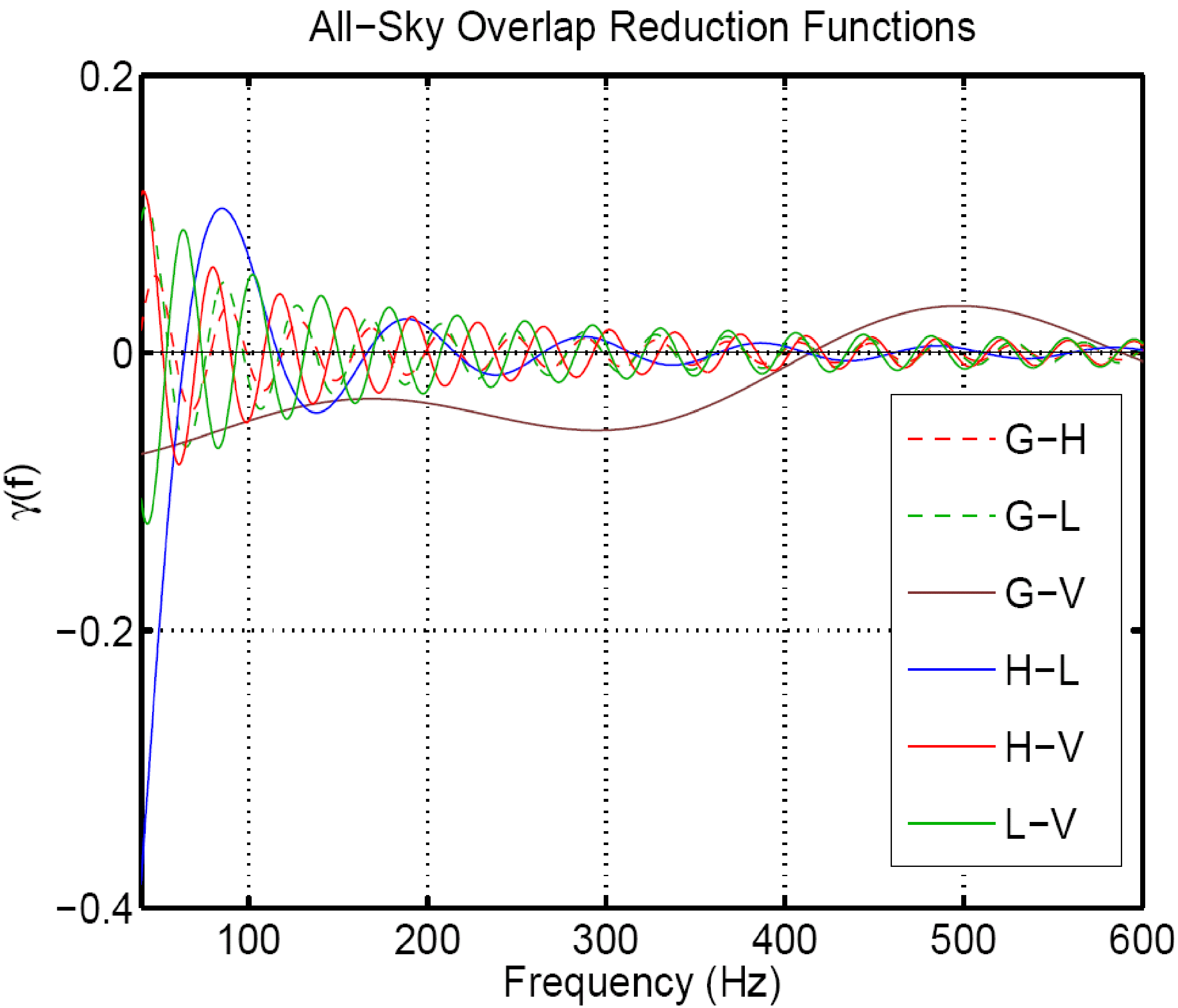
$$\left\langle \tilde{h}^\star_A(f)\tilde{h}_B(f')\right\rangle \propto \delta(f-f')S_h(f)\gamma_{AB}(f)$$

where the overlap reduction function γ_{AB} is defined by

$$\gamma_{AB}(f) = \frac{1}{F}\sum_P \frac{d\hat{\Omega}}{4\pi} D^{ij}_A \varepsilon^P_{ij}\left(\hat{\Omega}\right) D^{kl}_B \varepsilon^P_{kl}\left(\hat{\Omega}\right) e^{2\pi i f \hat{\Omega}\cdot\Delta\vec{r}}$$

- **Depends on distance (same features of strain correlations)**
- **Depends on orientation (via the overlap of detector tensors)**

ORF: EXAMPLES



Detection: basic idea

$$h_1 = s_1 + n_1$$

$$h_2 = s_2 + n_2$$

Basic idea: two detectors.

$$< h_1 h_2 > = < s_1 s_2 > + \cancel{< s_1 n_2 >} + \cancel{< n_1 s_2 >} + \cancel{< n_1 n_2 >}$$

1. noise not correlated with the signal

2. noise not correlated between the detectors (?)

What is the optimal detector?

Let's use the Neyman-Pearson lemma

Unpolarized, isotropic and stationary case. Optimal detector

- The data: a sequence of observations (k components, one for each detector)
- Probability distribution of the signal: a multivariate guassian distribution. In the stationary case it is diagonal in the frequency domain.
- $C=C_0$ in the hypothesis of no stochastic background:

  $x = n$
- $C=C_1$ in the hypothesis of the presence of a stochastic background:

  $x = n + h$

$$x_i^T = \left(x_i^{Virgo}, x_i^{Hanford}, \cdots, x_i^{Geo}\right)$$

$$dP = \mathcal{N} \prod_f \exp\left(-\frac{1}{2}\tilde{x}_f^+ C^{-1}(f)\tilde{x}_f\right) d^k\tilde{x}_f$$

$$C_0 = C_N = \begin{pmatrix} S_N^{Virgo} & 0 & \cdots & 0 \\ 0 & S_N^{Hanford} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & S_N^{Geo} \end{pmatrix}$$

$$C_1 = C_N + C_S$$

$$C_S = S_h \begin{pmatrix} 1 & \gamma_{V,H} & \cdots & \gamma_{V,G} \\ \gamma_{V,H} & 1 & \cdots & \gamma_{H,G} \\ \vdots & \vdots & \ddots & \cdots \\ \gamma_{V,G} & \gamma_{H,G} & \cdots & 1 \end{pmatrix}$$

Optimal statistic

- Neyman-Pearson recipe:

$$\frac{dP_1}{dP_0} = \exp\left[-\frac{1}{2}\int df\,\tilde{x}_f^{+}\left(\frac{1}{C_N + C_S} - \frac{1}{C_N}\right)\tilde{x}_f\right] > \eta$$

- Optimal statistic

$$Y = \lambda \int df \frac{\tilde{h}_1^{\star}(f)\tilde{h}_2(f)\gamma_{12}(f)S_h(f)}{S_{n,1}(f)S_{n,2}(f)}$$

SNR

$$SNR_Y^2 := \frac{\mu_Y^2}{\sigma_Y^2} = 2T \int_0^\infty S_h^2 \frac{\gamma_{12}(f)^2}{S_{n,1}(f) S_{n,2}(f)} df$$

It depends on:

- **theoretical PSD of stochastic background**
- **overlap reduction function**
- **noise PSD of the two detectors**
- **measurement time**

Many detectors

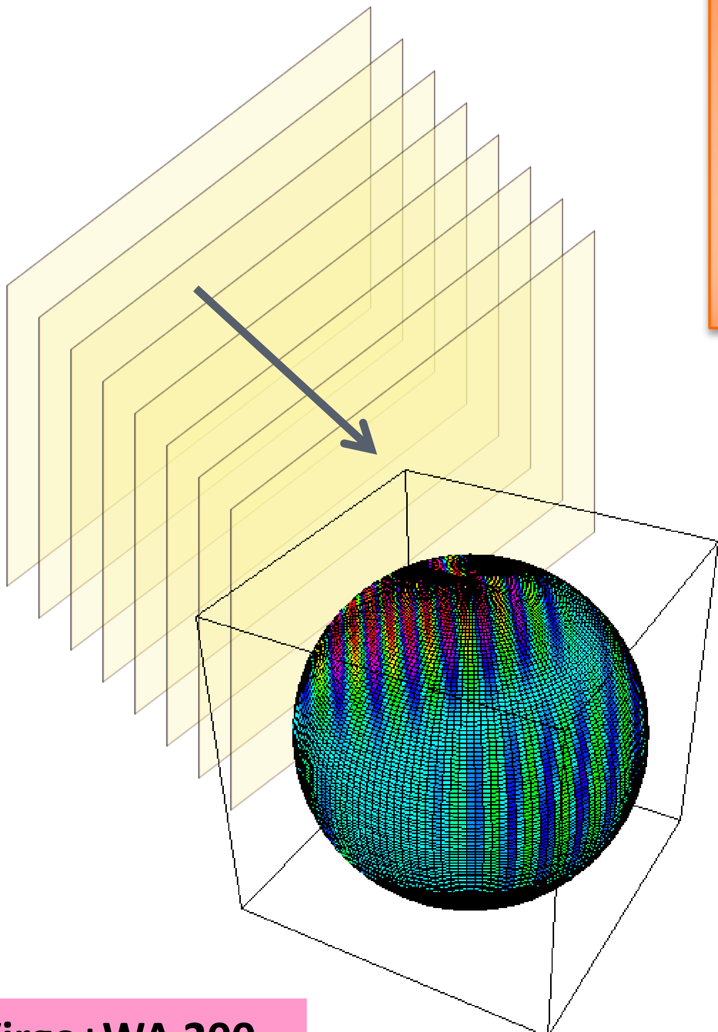
$$\frac{1}{\Omega_{gw}(\delta,\alpha)^2} = \frac{1}{2}\sum_{i\neq j}\frac{1}{\Omega_{gw}^{(i,j)}(\delta,\alpha)^2}$$

Gaussian signals: optimal detector is a combination of optimal correlations from each pair.

- **Not a large improvement factor, but**
- **Very important for ruling out spurious effects**

Targeted search: reconstruct a map of the gravitational wave uminosity in the sky

- Correct the direction-dependent moduation
- Cross-Correlate
- At least 3 detectors needed to close the inverse problem.
- Angular resolution limited by λ/D



Virgo+WA 200 Hz

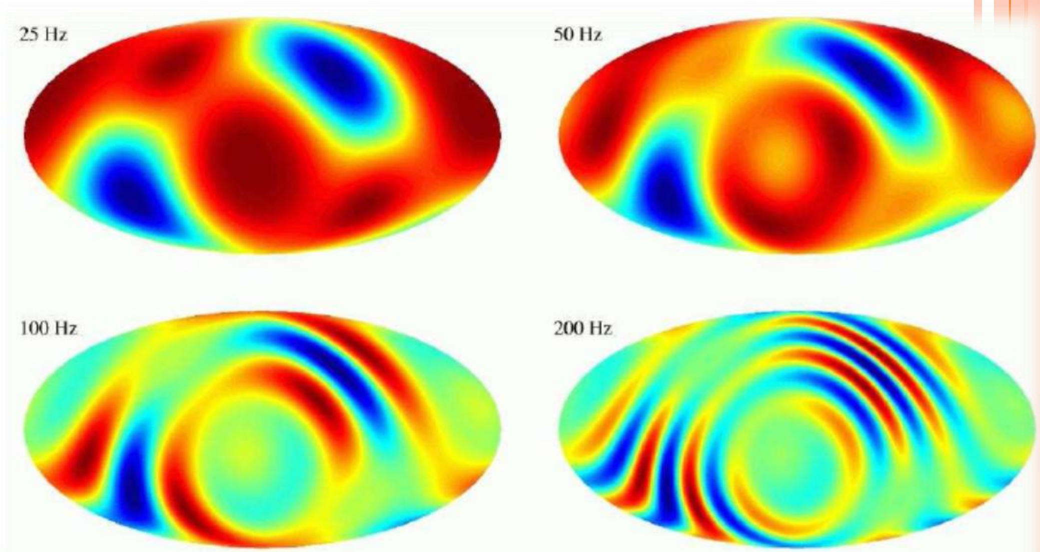


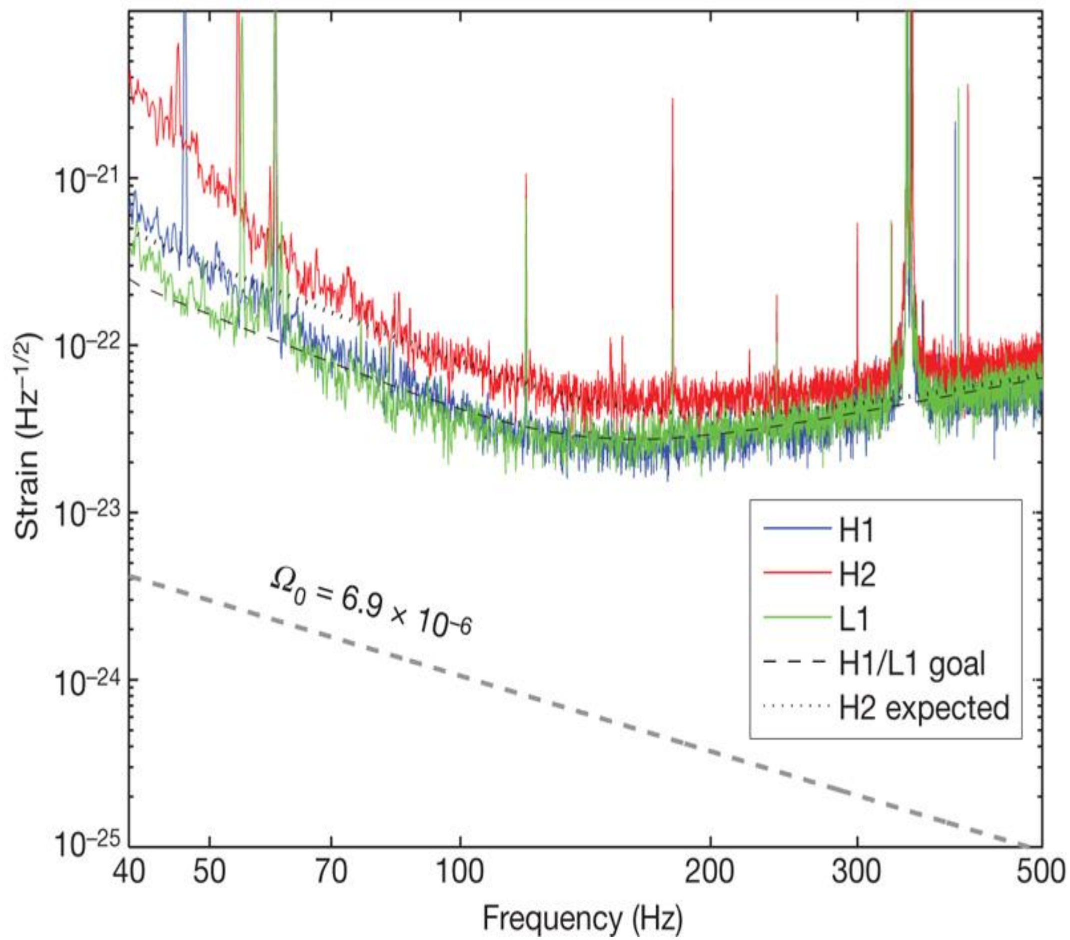
Figure 1. The antenna pattern for the cross-correlated LIGO detectors in the Earth-fixed frame, $\mathcal{F}(\bar{\theta}, \bar{\phi}, f)$, with $g_I = 0$ and $f = 25, 50, 100$ and 200 Hz.

Cornish N.J., 2001, "Mapping the Gravitational Wave Background", CQG, 18, 4277; astro-ph/0105374

CURRENT UPPER LIMIT FOR ISOTROPIC BACKGROUND

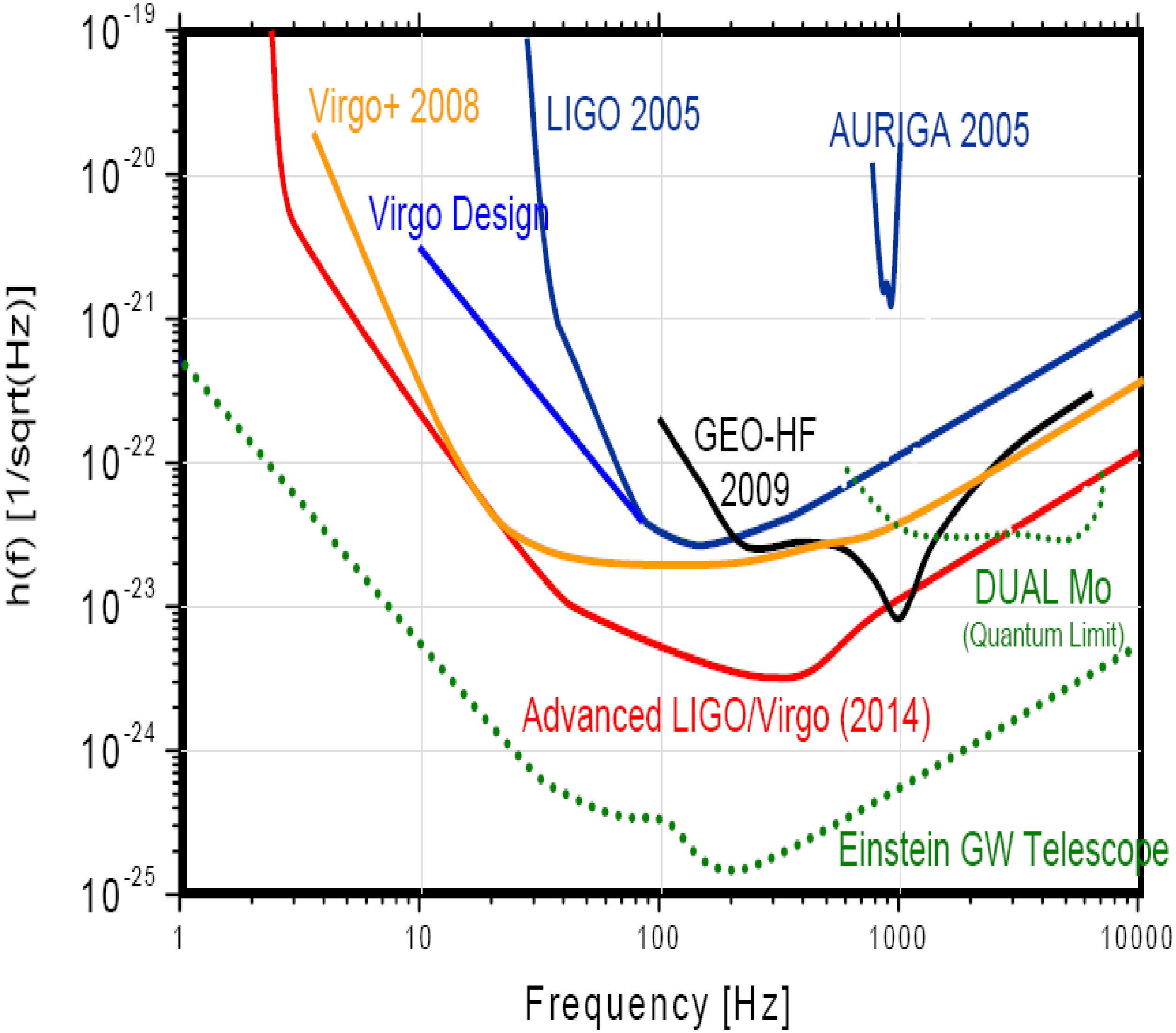
- Flat spectrum
- Factor 100 lower than sensitivity: *the SNR increases with the integration time*

Sensitivities of LIGO interferometers.



The LIGO Scientific Collaboration & The Virgo Collaboration *Nature* **460**, 990-994 (2009) doi:10.1038/nature08278

FUTURE PERSPECTIVES



References: reviews

1. **Bruce Allen . The Stochastic gravity wave background: Sources and detection. gr-qc/9604033**
   A world-wide effort is now underway to build gravitational wave detectors based on highly-sensitive laser interferometers. When data from detectors at different sites is properly combined, it will permit highly-sensitive searches for a stochastic background of relic gravitational radiation. These lectures (from the Les Houches School in October 1995) review the current status of this program, and discuss the methods by which data from different detectors can be used to make measurements of, or place limits on, a stochastic background. They also review possible cosmological sources and their potential detectability.
2. **Michele Maggiore. Gravitational wave experiments and early universe cosmology. Phys.Rept. 331 (2000) 283-367**
   Gravitational-wave experiments with interferometers and with resonant masses can search for stochastic backgrounds of gravitational waves of cosmological origin. We review both experimental and theoretical aspects of the search for these backgrounds. We give a pedagogical derivation of the various relations that characterize the response of a detector to a stochastic background. We discuss the sensitivities of the large interferometers under constructions (LIGO, VIRGO, GEO600, TAMA300, AIGO) or planned (Avdanced LIGO, LISA) and of the presently operating resonant bars, and we give the sensitivities for various two-detectors correlations. We examine the existing limits on the energy density in gravitational waves from nucleosynthesis, COBE and pulsars, and their effects on theoretical predictions. We discuss general theoretical principles for order-of-magnitude estimates of cosmological production mechanisms, and then we turn to specific theoretical predictions from inflation, string cosmology, phase transitions, cosmic strings and other mechanisms. We finally compare with the stochastic backgrounds of astrophysical origin.

References: detection (I)

1. **Bruce Allen, Joseph D. Romano . Detecting a stochastic background of gravitational radiation: Signal processing strategies and sensitivities. Phys.Rev.D59:102001,1999.**
   We analyze the signal processing required for the optimal detection of a stochastic background of gravitational radiation using laser interferometric detectors. Starting with basic assumptions about the statistical properties of a stochastic gravity-wave background, we derive expressions for the optimal filter function and signal-to-noise ratio for the cross-correlation of the outputs of two gravity-wave detectors. Sensitivity levels required for detection are then calculated. Issues related to: (i) calculating the signal-to-noise ratio for arbitrarily large stochastic backgrounds, (ii) performing the data analysis in the presence of nonstationary detector noise, (iii) combining data from multiple detector pairs to increase the sensitivity of a stochastic background search, (iv) correlating the outputs of 4 or more detectors, and (v) allowing for the possibility of correlated noise in the outputs of two detectors are discussed. We briefly describe a computer simulation which mimics the generation and detection of a simulated stochastic gravity-wave signal in the presence of simulated detector noise. Numerous graphs and tables of numerical data for the five major interferometers (LIGO-WA, LIGO-LA, VIRGO, GEO-600, and TAMA-300) are also given. The treatment given in this paper should be accessible to both theorists involved in data analysis and experimentalists involved in detector design and data acquisition.

2. **Bruce Allen, Adrian C. Ottewill . Detection of anisotropies in the gravitational wave stochastic background. Phys.Rev.D56:545-563,1997.**
   By correlating the signals from a pair of gravitational-wave detectors, one can undertake sensitive searches for a stochastic background of gravitational radiation. If the stochastic background is anisotropic, then this correlated signal varies harmonically with the earth's rotation. We calculate how the harmonics of this varying signal are related to the multipole moments which characterize the anisotropy, and give a formula for the signal-to-noise ratio of a given harmonic. The specific case of the two LIGO (Laser Interferometric Gravitational Observatory) detectors, which will begin operation around the year 2000, is analyzed in detail. We consider two possible examples of anisotropy. If the gravitational-wave stochastic background contains a dipole intensity anisotropy whose origin (like that of the Cosmic Background Radiation) is motion of our local system, then that anisotropy will be observable by the advanced LIGO detector (with 90% confidence in one year of observation) if $\Omega_{gw} > 5.3 \times 10^{-8} h_{100}^{-2}$. We also study the signal produced by stochastic sources distributed in the same way as the luminous matter in the galactic disk, and in the same way as the galactic halo. The anisotropy due to sources distributed as the galactic disk or as the galactic halo will be observable by the advanced LIGO detector (with 90% confidence in one year of observation) if $\Omega_{gw} > 1.8 \times 10^{-10} h_{100}^{-2}$ or $\Omega_{gw} > 6.7 \times 10^{-8} h_{100}^{-2}$, respectively.

References: Detection (II)

1. **LIGO Scientific and VIRGO Collaborations (B.P. Abbott et al.). An Upper Limit on the Stochastic Gravitational-Wave Background of Cosmological Origin. Nature 460 (2009) 990 .**
A stochastic background of gravitational waves is expected to arise from a superposition of a large number of unresolved gravitational-wave sources of astrophysical and cosmological origin. It is expected to carry unique signatures from the earliest epochs in the evolution of the universe, inaccessible to the standard astrophysical observations. Direct measurements of the amplitude of this background therefore are of fundamental importance for understanding the evolution of the universe when it was younger than one minute. Here we report direct limits on the amplitude of the stochastic gravitational-wave background using the data from a two-year science run of the Laser Interferometer Gravitational-wave Observatory (LIGO). Our result constrains the energy density of the stochastic gravitational-wave background normalized by the critical energy density of the universe, in the frequency band around 100 Hz, to be less than 6.9 x 10^{-6} at 95% confidence. The data rule out models of early universe evolution with relatively large equation-of-state parameter, as well as cosmic (super)string models with relatively small string tension that are favoured in some string theory models. This search for the stochastic background improves upon the indirect limits from the Big Bang Nucleosynthesis and cosmic microwave background at 100 Hz.

2. **John T Whelan**
Stochastic gravitational wave measurements with bar detectors: Dependence of response on detector orientation.
Class.Quant.Grav.23:1181-1192,2006.
The response of a cross-correlation measurement to an isotropic stochastic gravitational-wave background depends on the observing geometry via the overlap reduction function. If one of the detectors being correlated is a resonant bar whose orientation can be changed, the response to stochastic gravitational waves can be modulated. I derive the general form of this modulation as a function of azimuth, both in the zero-frequency limit and at arbitrary frequencies. Comparisons are made between pairs of nearby detectors, such as LIGO Livingston-ALLEGRO, Virgo-AURIGA, Virgo-NAUTILUS, and EXPLORER-AURIGA, with which stochastic cross-correlation measurements are currently being performed, planned, or considered.

REFERENCES: INTENSITY MAP OF STOCHASTIC BACKGROUND INTENSITY

1. **Sanjit Mitra, Sanjeev Dhurandhar, Tarun Souradeep, Albert Lazzarini, Vuk Mandic, Sukanta Bose, Stefan Ballmer.**
   Gravitational wave radiometry: Mapping a stochastic gravitational wave background.
   Phys.Rev.D77:042002,2008.
   The problem of the detection and mapping of a stochastic gravitational wave background (SGWB), either of cosmological or astrophysical origin, bears a strong semblance to the analysis of CMB anisotropy and polarization. The basic statistic we use is the cross-correlation between the data from a pair of detectors. In order to `point' the pair of detectors at different locations one must suitably delay the signal by the amount it takes for the gravitational waves (GW) to travel to both detectors corresponding to a source direction. Then the raw (observed) sky map of the SGWB is the signal convolved with a beam response function that varies with location in the sky. We first present a thorough analytic understanding of the structure of the beam response function using an analytic approach employing the stationary phase approximation. The true sky map is obtained by numerically deconvolving the beam function in the integral (convolution) equation. We adopt the maximum likelihood framework to estimate the true sky map that has been successfully used in the broadly similar, well-studied CMB map making problem. We numerically implement and demonstrate the method on simulated (unpolarized) SGWB for the radiometer consisting of the LIGO pair of detectors at Hanford and Livingston. We include `realistic' additive Gaussian noise in each data stream based on the LIGO-I noise power spectral density. The extension of the method to multiple baselines and polarized GWB is outlined. In the near future the network of GW detectors, including the Advanced LIGO and Virgo detectors that will be sensitive to sources within a thousand times larger spatial volume, could provide promising data sets for GW radiometry.

REFERENCES: CORRELATED NOISE BETWEEN DETECTORS

1. **Matthew R. Adams, Neil J. Cornish**
Discriminating between a Stochastic Gravitational Wave Background and Instrument Noise.
Phys.Rev.D82:022002,2010.
The detection of a stochastic background of gravitational waves could significantly impact our understanding of the physical processes that shaped the early Universe. The challenge lies in separating the cosmological signal from other stochastic processes such as instrument noise and astrophysical foregrounds. One approach is to build two or more detectors and cross correlate their output, thereby enhancing the common gravitational wave signal relative to the uncorrelated instrument noise. When only one detector is available, as will likely be the case with the Laser Interferometer Space Antenna (LISA), alternative analysis techniques must be developed. Here we show that models of the noise and signal transfer functions can be used to tease apart the gravitational and instrument noise contributions. We discuss the role of gravitational wave insensitive "null channels" formed from particular combinations of the time delay interferometry, and derive a new combination that maintains this insensitivity for unequal arm length detectors. We show that, in the absence of astrophysical foregrounds, LISA could detect signals with energy densities as low as $\Omega_{\rm gw} = 6 \times 10^{-13}$ with just one month of data. We describe an end-to-end Bayesian analysis pipeline that is able to search for, characterize and assign confidence levels for the detection of a stochastic gravitational wave background, and demonstrate the effectiveness of this approach using simulated data from the third round of Mock LISA Data Challenges.

References: Astrophysical stochastic background

1. **Tania Regimbau. The astrophysical gravitational wave stochastic background Res.Astron.Astrophys.11:369-390,2011.**
   A gravitational wave stochastic background of astrophysical origin may have resulted from the superposition of a large number of unresolved sources since the beginning of stellar activity. Its detection would put very strong constrains on the physical properties of compact objects, the initial mass function or the star formation history. On the other hand, it could be a 'noise' that would mask the stochastic background of cosmological origin. We review the main astrophysical processes able to produce a stochastic background and discuss how it may differ from the primordial contribution by its statistical properties. Current detection methods are also presented.
2. **T. Regimbau, V. Mandic. Astrophysical Sources of Stochastic Gravitational-Wave Background. Class.Quant.Grav. 25 (2008) 184018.**
   We review the spectral properties of stochastic backgrounds of astrophysical origin and discuss how they may differ from the primordial contribution by their statistical properties. We show that stochastic searches with the next generation of terrestrial interferometers could put interesting constrains on the physical properties of astrophysical populations, such as the ellipticity and magnetic field of magnetars, or the coalescence rate of compact binaries.
3. **Raffaella Schneider, Valeria Ferrari, Sabino Matarrese . Stochastic backgrounds of gravitational waves from cosmological populations of astrophysical sources. astro-ph/9903470.**
   Astrophysical sources of gravitational radiation are likely to have been formed since the beginning of star formation. Realistic source rates of formation throughout the Universe have been estimated from an observation-based determination of the star formation rate density evolution. Both the radiation emitted during the collapse to black holes and the spin-down radiation, induced by the r-mode instability, emitted by hot, young rapidly rotating neutron stars have been considered. We have investigated the overall signal produced by the ensemble of sources exploring the parameter space and discussing its possible detectability.

Reference: non Gaussian stochastic background

1. **Steve Drasco, Eanna E. Flanagan . Detection methods for nonGaussian gravitational wave stochastic backgrounds. Phys.Rev.D67:082003,2003.**
   We address the issue of finding an optimal detection method for a discontinuous or intermittent gravitational wave stochastic background. Such a signal might sound something like popcorn popping. We derive an appropriate version of the maximum likelihood detection statistic, and compare its performance to that of the standard cross-correlation statistic both analytically and with Monte Carlo simulations. The maximum likelihood statistic performs better than the cross-correlation statistic when the background is sufficiently non-Gaussian. For both ground and space based detectors, this results in a gain factor, ranging roughly from 1 to 3, in the minimum gravitational-wave energy density necessary for detection, depending on the duty cycle of the background. Our analysis is exploratory, as we assume that the time structure of the events cannot be resolved, and we assume white, Gaussian noise in two collocated, aligned detectors. Before this detection method can be used in practice with real detector data, further work is required to generalize our analysis to accommodate separated, misaligned detectors with realistic, colored, non-Gaussian noise.

2. **Bruce Allen, Jolien D.E. Creighton, Eanna E. Flanagan , Joseph D. Romano**
   Robust statistics for deterministic and stochastic gravitational waves in nonGaussian noise. 1. Frequentist analyses. Phys.Rev.D65:122002,2002.
   Gravitational wave detectors will need optimal signal-processing algorithms to extract weak signals from the detector noise. Most algorithms designed to date are based on the unrealistic assumption that the detector noise may be modeled as a stationary Gaussian process. However most experiments exhibit a non-Gaussian ``tail'' in the probability distribution. This ``excess'' of large signals can be a troublesome source of false alarms. This article derives an optimal (in the Neyman-Pearson sense, for weak signals) signal processing strategy when the detector noise is non-Gaussian and exhibits tail terms. This strategy is robust, meaning that it is close to optimal for Gaussian noise but far less sensitive than conventional methods to the excess large events that form the tail of the distribution. The method is analyzed for two different signal analysis problems: (i) a known waveform (e.g., a binary inspiral chirp) and (ii) a stochastic background, which requires a multi-detector signal processing algorithm. The methods should be easy to implement: they amount to truncation or clipping of sample values which lie in the outlier part of the probability distribution.

3. **Bruce Allen, Jolien D.E. Creighton, Eanna E. Flanagan, Joseph D. Romano . Robust statistics for deterministic and stochastic gravitational waves in nonGaussian noise. 2. Bayesian analyses. Phys.Rev.D67:122002,2003.**
   In a previous paper we derived a set of near-optimal signal detection techniques for gravitational wave detectors whose noise probability distributions contain non-Gaussian tails. The methods modify standard methods by truncating sample values which lie in those non-Gaussian tails. The methods were derived, in the frequentist framework, by minimizing false alarm probabilities at fixed false detection probability in weak signal limit. For stochastic signals, the resulting statistic consisted of a sum of an auto-correlation term and a cross-correlation term; it was necessary to discard by hand the auto-correlation term to obtain the correct, generalized cross-correlation statistic. In the present paper, we present an alternative Bayesian derivation of the same signal detection techniques. We compute the probability that a signal is present in the data, in the limit where the signal-to-noise ratio squared per frequency bin is small, where the integrated signal-to-noise ratio is large compared to one, and where the total probability in the non-Gaussian tail part of the noise distribution is small. We show that, for each model considered, the resulting probability is to a good approximation a monotonic function of the detection statistic derived in the previous paper. Moreover, for stochastic signals, the new Bayesian derivation automatically eliminates the problematic auto-correlation term.

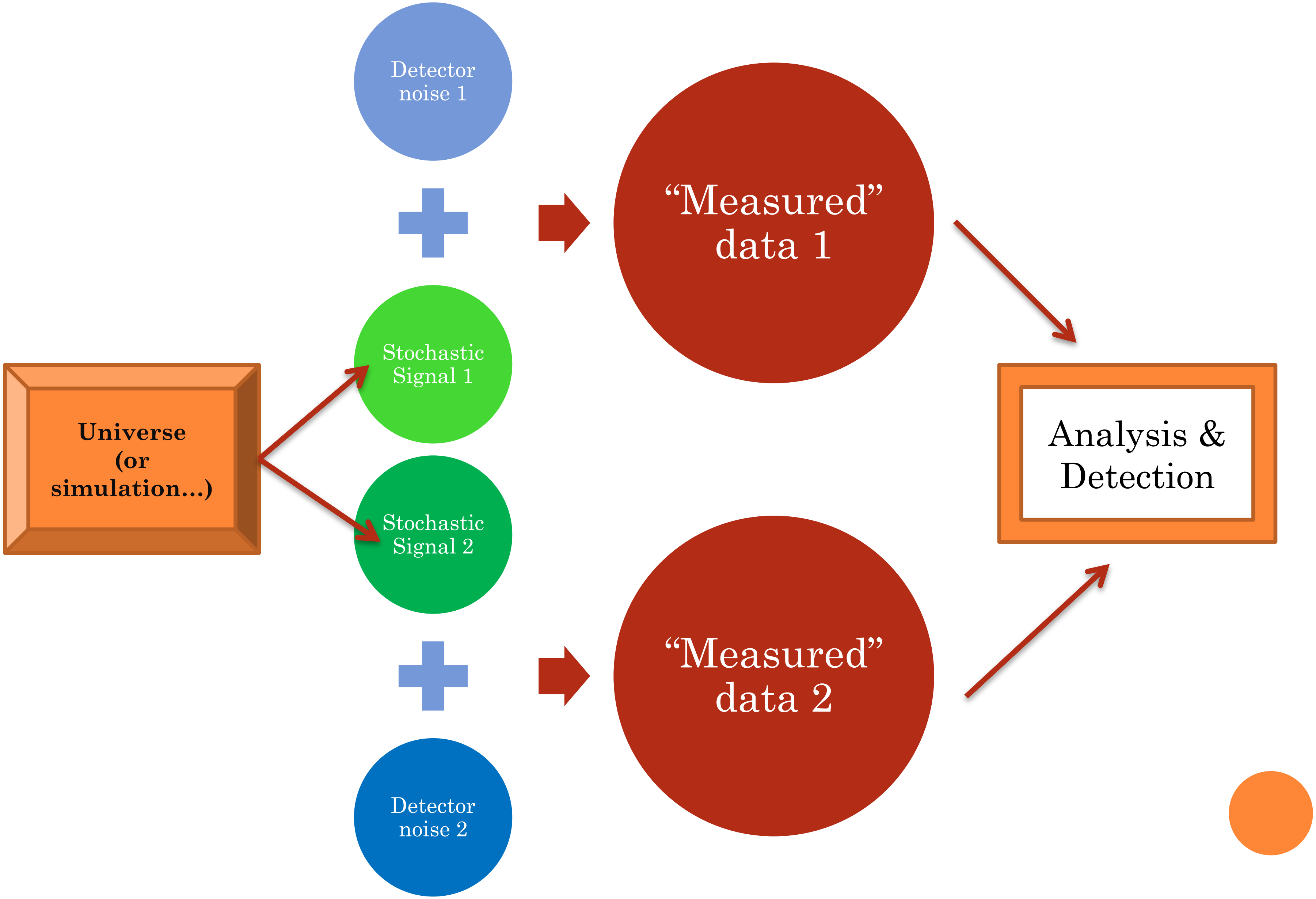
Pratical Session

Information

Basic istructions to start a session

1. Login
2. Prepare the environment:
   > *source /data/procdata/bufferv21/SB/VESF2011/vesfenv.tcsh*
3. Get the initialization files
   > *getinifiles*
4. Get the data files
   > *getdatafiles*

GENEALOGY OF DATA



Data files available: Signal & Noise

- «Hanford»:
  - H1a.ffl
  - H1aS.ffl (surrogate)
  - H1b.ffl
  - H1bS.ffl (surrogate)
- «Virgo»:
  - V1a.ffl
  - V1b.ffl

Set a: low SNR

Set b: high SNR

The command *getdatafiles* transfer the files in the current directory.

Other data:

- Signal only:
  - signalH1.ffl
  - signalV1.ffl
- Noise only:
  - H1noise.ffl
  - H1noiseS.ffl
  - V1noise.ffl
- Scripts:
  - PSD.py
  - Coherence.py
  - Pipeline.py

PSD.INI

- [DATA]
- filename: V1b.ffl
- channel: V1:strain
- start: 871385951
- length: 1
- averages: 1000
- window: Welch
- update: 10
- plot: true
- save: true
- out: STRAIN_psd%d.dat

PSD.py

Coherence.ini

- [DATA]
- filename1: virgoData/ffl/raw.ffl
- channel1: Pr_B1_Acp
- gps1: 849165179
- length1: 12
- filename2: /virgoData/ffl/raw.ffl
- channel2: Pr_B1_Acp
- gps2: 849165179
- length2: 12
- averages: 60
- window: Welch
- type: Coherence
- update: 100
- plot: true
- save: true
- plotTemplate: startup.agr
- plotFinal: coherence.agr

Pipeline.ini

- [DETECTOR1]
- Name: Virgo
- Filename: V1b.ffl
- Channel: V1:strain
- GpsStart: 849165179
- [DETECTOR2]
- Name: LHO
- Filename: H1b.ffl
- Channel: H1:strain
- GpsStart: 849165179
- [ANALYSIS]
- BufferSize: 60
- Samples: 1440
- Beta: 2.0
- UseOverlapReductionFunction:
- falseMaskStart: 0.0 1000.0
- MaskStop: 10.0 2000.0