Data Analysis:
Coalescing Binaries
(a brief introduction, mostly qualitative)

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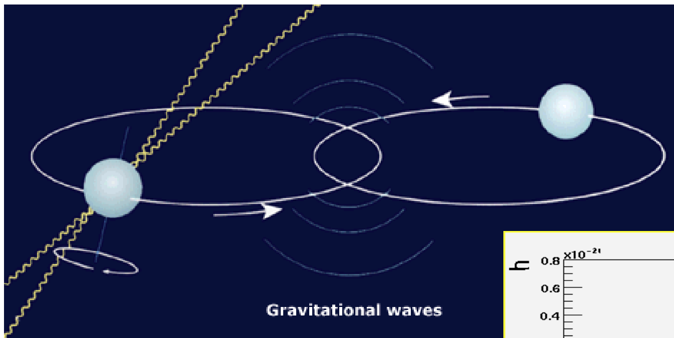
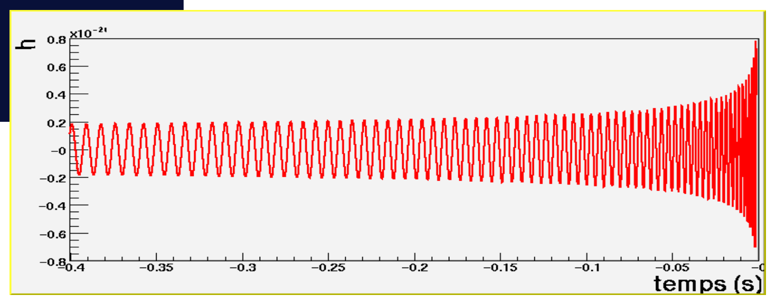
Outline

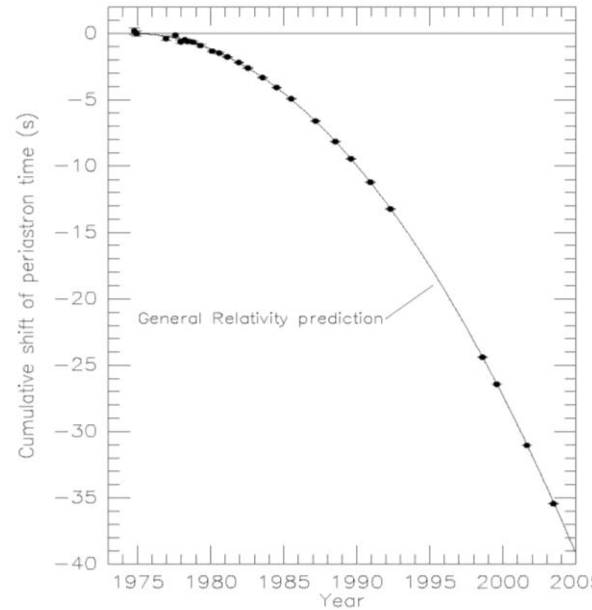
- A reminder about what we are looking for
  - » The source, the waveform
- The basic search technique
  - » Matched filtering
- Exploring the parameter space
- Real life: dealing with background
  - » Coincidences, vetoes
- Network analysis
- A brief review of LIGO-Virgo CBC searches

The target sources

- Final evolution stage of compact binary systems
  - » Systems like PSR1913+16 reaching coalescence of the two stars







- System may involve
  - » Neutron stars
  - » Black holes
    - – For ground based detectors, stellar mass black holes
    - – Advanced detectors: up to intermediate mass black holes
    - – Super-massive BH: lower frequency, space based detectors

$$f_{\mathrm{ISCO}} = \frac{2.8 M_\odot}{M} \quad 1600\ \mathrm{Hz}$$

What makes CB promising sources?

- We know “a lot” about the sources
  - » Such systems do exist
    - – Although rates are uncertain and low…
  - » The emitted waveform is known with some accuracy
- A nice laboratory to study General Relativity
  - » Confront waveform prediction with observation
  - » Study GR at work in the strong field regime
- A nice tool for astrophysics and cosmology
  - » Parameters of the system can be extracted
  - » CB are standard candles: source distance can be measured
    - – Opens the possibility of measuring the Hubble constant

    [Ref.1]
  - » Are short gamma ray bursts associated to coalescing binaries?

Rare events: BNS systems

- Galactic rate
  - » CB rate in the Galaxy inferred from known systems, expected to reach coalescence in a time less than the age of the Universe
  - » Only 3 such systems known today (including PSR 1913+16)
  - » Estimate dominated by most recently discovered system (PSR J0737+3039)
  - » Estimate depends on the modeled Galactic distribution of neutron stars
  - » R ~ 1 – 1000 $MWEG^{-1}$ Myr^{-1}, realistic estimate R ~100 $MWEG^{-1}$ Myr^{-1}

[Ref.2]

- Detected rate
  - » Rate of detected events depends on number of galaxies probed by the detector
  - » Related to detector horizon distance (distance at which an optimally located and oriented source would produce a SNR of 8)
    - – For initial detectors ($D_{horizon}$ ~ 30 Mpc)
      N ~ 2 .10^{-4} – 2 .10^{-1} yr^{-1}, most probable N ~ 1 / (50 yr)
    - – For advanced detectors (assuming 15 times improved horizon distance)
      most probable N ~ 40 / yr

Rare events: BH-NS & BH-BH

- No known system involving a black hole
  - » Rely on stellar evolution models to predict rate
  - » Galactic coalescence rate smaller for BH-NS or BH-BH systems than for NS-NS systems
  - » Systems with BH can be seen up to larger distances
  - ⇒ Overall detected rate larger ??
  - » Initial detectors

[Ref.3]

$$N_{BHBH} \sim 7 \,.\, 10^{-3}\ yr^{-1}$$

$$N_{NSBH} \sim 4 \,.\, 10^{-3}\ yr^{-1}$$

  - » Advanced detectors

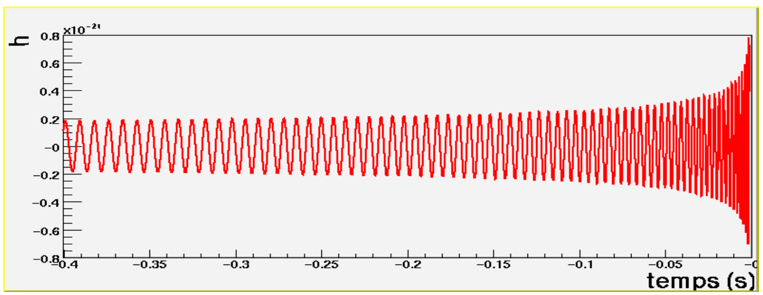
$$N_{BHBH} \sim 20\ yr^{-1}$$

$$N_{NSBH} \sim 10\ yr^{-1}$$

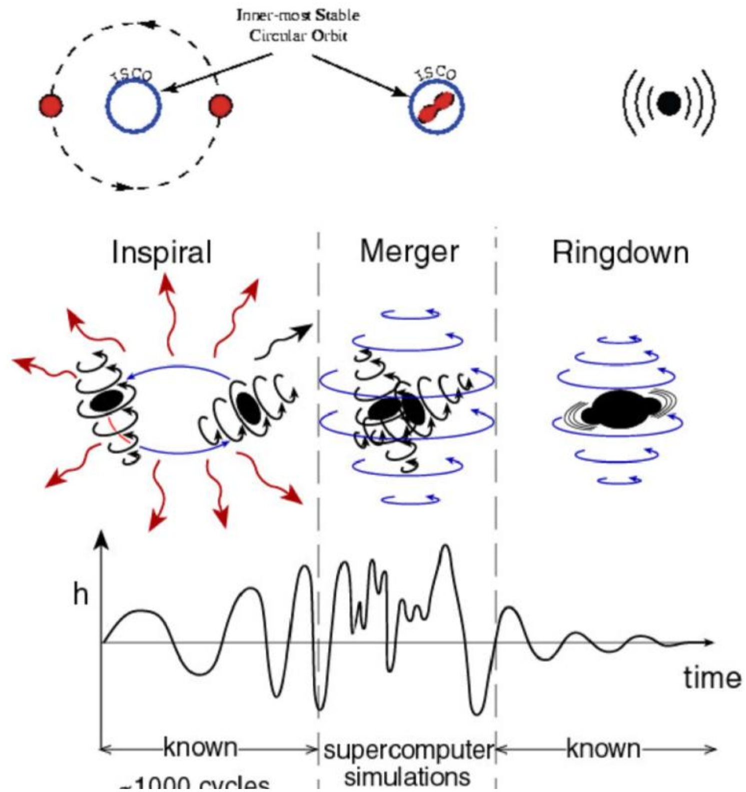
Large uncertainties on those numbers!!

Phases of the evolution

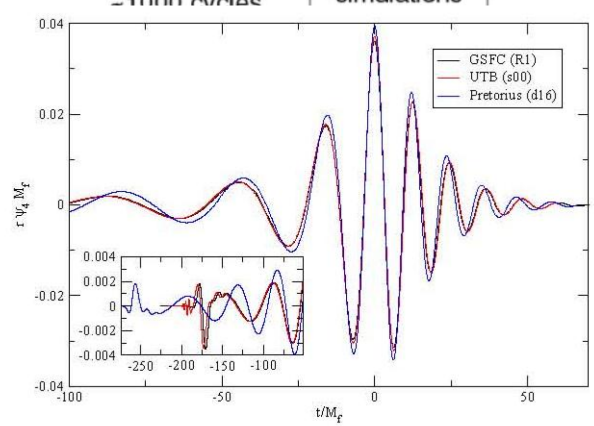
- ## Inspiral phase
  - The realm of post-Newtonian expansions
  - Accurately known chirp, at least for those light enough systems well described by adiabatic models



$$\text{duration} \sim 34 \left(\frac{\mathcal{M}}{M_\odot}\right)^{-5/3} \left(\frac{f_0}{40\ \text{Hz}}\right)^{-8/3} \text{s}$$



- ## Plunge, merger and ringdown
  - The realm of numerical relativity
  - Duration << 1 s
  - Not crucial for detection unless it is the only part of the signal within the bandwidth of the detector



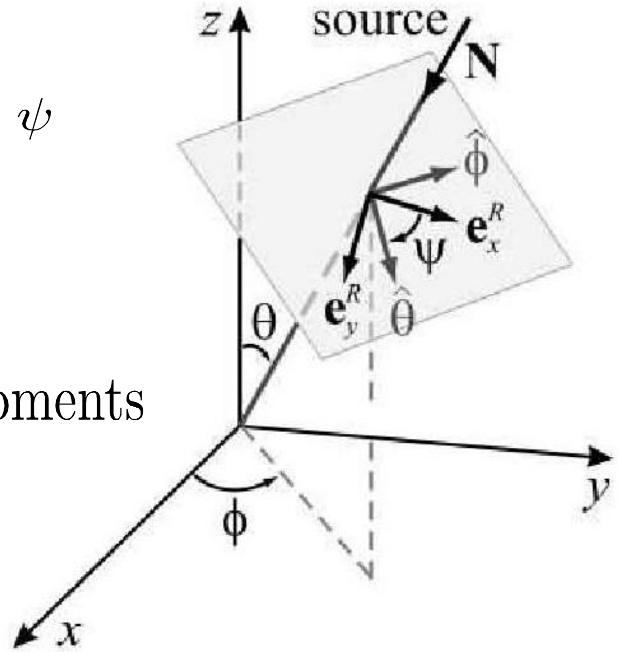
The waveform (I)

$$\mathrm{h(t)} = \mathrm{F}_+(\theta,\phi,\psi)\ h_+(t) + F_\times(\theta,\phi,\psi)\ h_\times(t)$$

F_+ and $F_\times$: detector response functions
depend on sky location (θ,ϕ) and polarization angle ψ

$$F_+ = -\tfrac{1}{2}(1+\cos^2\theta)\cos 2\phi\cos 2\psi - \cos\theta\sin 2\phi\sin 2\psi$$

$$F_\times = \tfrac{1}{2}(1+\cos^2\theta)\cos 2\phi\sin 2\psi - \cos\theta\sin 2\phi\cos 2\psi$$



h_+ and $h_\times$ are obtained from post-Newtonian developments

Up to 2.5PN order in amplitude:

$$\mathrm{h(t)} = \sum_{k=1}^{N}\sum_{m=0}^{5}\ A_{k,m/2}(t)\ \cos(k\ \varphi(t) + \varphi_{k,m/2})$$

with $A_{k,m/2}(t)\ \propto (2\pi M f(t))^{(m+2)/3}$

» Usual searches use restricted waveforms, namely waveforms where all terms with k ≠ 2 are neglected: other harmonics of the orbital frequency are ignored

» OK from the detection point of view, at least for initial detectors

» May reduce the accuracy of parameter estimation, especially for high mass systems

[Ref.21]

The waveform (II)

The restricted waveform at the detector can be written:

$$h(t) = \frac{1\ \mathrm{Mpc}}{D_{\mathrm{eff}}}\ A(t)\ \cos(\varphi(t) - \varphi_0)$$

$$\text{with } D_{\mathrm{eff}} = \frac{D}{\sqrt{F_+^2(1+\cos^2\iota)^2/4 + F_\times^2\cos^2\iota}} \text{ the effective distance}$$

(distance of an optimally located and oriented source that would produce the same signal strength)

$$A(t) = A\ f(t)^{2/3}$$

At Newtonian order:

$$f(t) = f_0\ (1 - \frac{t}{\tau_0})^{-3/8} \qquad \varphi(t) = \frac{16\pi f_0 \tau_0}{5}\left[1 - \left(\frac{f}{f_0}\right)^{-5/3}\right]$$

$$\tau_0 = \frac{5}{256}\mathcal{M}^{-5/3}(\pi f_0)^{-8/3} \text{ time from frequency } f_0 \text{ to coalescence}$$

$\mathcal{M}$ is called the chirp mass

$$\mathcal{M} = \mu^{3/5}M^{2/5} \quad M = m_1 + m_2 \quad \mu = m_1 m_2/M$$
$$= \eta^{3/5}M \qquad \eta = \mu/M$$

The waveform (III)

- PN developments [Ref.4]
  - » Known up to order PN2.5 for the amplitude and order PN3.5 for the phase
    - – Most searches use restricted PN2 waveforms
    - – Good enough for detection, may cost some accuracy in parameter estimation
  - » Spin effects appear from order PN1.5 (spin-orbit) and PN2 (spin-spin)
    - – Expected to be negligible for NS, may be significant for BH
- PN developments become inaccurate for high mass systems
  - » A variety of alternative waveforms exist
    - – Padé approximants, EOB (effective one body)… [Ref.5, 6]
  - » Detection template families can also be considered
    - – Phenomenological templates grasping the features of the different models
      - BCV [Ref.7]
    - – Provide good detection efficiency, but may suffer from high false alarm probability in real life, due to the inadequacy of signal based vetoes with such waveforms

Matched filtering (I)

» Construct a filtered signal

$$S = \int_{-\infty}^{\infty} h(t)Q(t)dt$$

detector output

filter chosen to optimize the signal to noise ratio (SNR)

» *S* can also be written in the frequency domain

$$S = \int_{-\infty}^{\infty} \tilde{h}(f)\tilde{Q}^*(f)df$$

» If the detector output is noise + some signal

$$h(t) = n(t) + C(t) \text{ with } C(t) = \alpha T(t - t_0)$$

$T(t)$: normalized expected signal entering detector bandwidth at time $t = 0$

» The expectation value of the signal *S* is

$$< S >= \int_{-\infty}^{\infty} < \tilde{h}(f) > \tilde{Q}^*(f)df = \int_{-\infty}^{\infty} \tilde{C}(f)\tilde{Q}^*(f)df$$

Matched filtering (II)

» The noise is defined as:

$$N = S - < S > = \int_{-\infty}^{\infty} \tilde{n}(f)\tilde{Q}^*(f)df$$

$$< N >= 0 \quad \text{but} \quad < N^2 >= \int_0^{\infty} S_h(f)|\tilde{Q}(f)|^2 df$$

where $S_h(f)$ is the one-sided noise power spectrum of the detector:

$$< \tilde{n}(f)\tilde{n}^*(f') >= \frac{1}{2}\, S_h(|f|)\, \delta(f - f')$$

» We can define an inner product

$$(A, B) = \int_{-\infty}^{\infty} \tilde{A}(f)\tilde{B}^*(f)S_h(|f|)df$$

and rewrite $< S >= (\frac{\tilde{C}}{S_h}, \tilde{Q})$ and $< N^2 >= \frac{1}{2}(\tilde{Q}, \tilde{Q})$

» What is the optimal filter Q maximizing $SNR^2 = \frac{<S>^2}{<N^2>} = 2\frac{(\frac{\tilde{C}}{S_h},\tilde{Q})^2}{(\tilde{Q},\tilde{Q})}$?

use property $(A, B)^2 \leq (A, A)(B, B)$

$$(A, B)^2 = (A, A)(B, B) \text{ only if A proportional to B}$$

Matched filtering (III)

» We choose $\tilde{Q}(f) \propto \frac{\tilde{C}(f)}{S_h(|f|)} = \alpha \, \frac{\tilde{T}(f)}{S_h(|f|)} e^{2\pi i f t_0}$

» Signal S for arrival time offset t_0 is given by

$$S = \int_{-\infty}^{\infty} \tilde{h}(f)\tilde{Q}^*(f) df$$

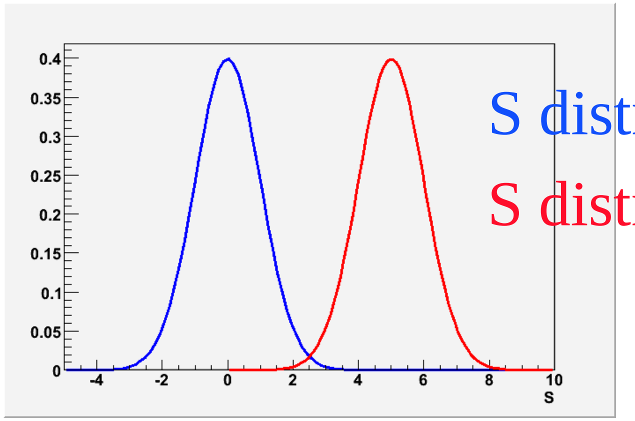
$$= \alpha \int_0^{\infty} \frac{\tilde{h}(f)\tilde{T}^*(f)}{S_h(f)} e^{-2\pi i f t_0} \, df$$

Fourier transform of S

S can be easily obtained for all arrival times t_0 by means of an FFT

» The optimal signal to noise ratio is: $SNR^2 = 2\alpha^2 (\frac{\tilde{T}}{S_h}, \frac{\tilde{T}}{S_h})$

If T is normalized such that $(\frac{\tilde{T}}{S_h}, \frac{\tilde{T}}{S_h}) = \frac{1}{2}$ then $< N^2 >= 1$ and $SNR^2 = \alpha^2$

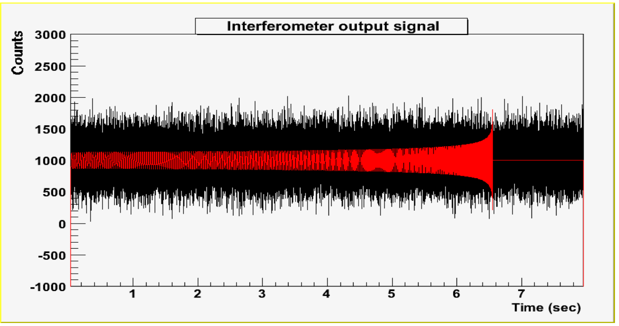


S distribution for noise

S distribution for signal with $\alpha = 5$

Matched filtering in practice (I)

- The FFT allows to extract S for all possible arrival times
  - » Easy to maximize SNR over t_0



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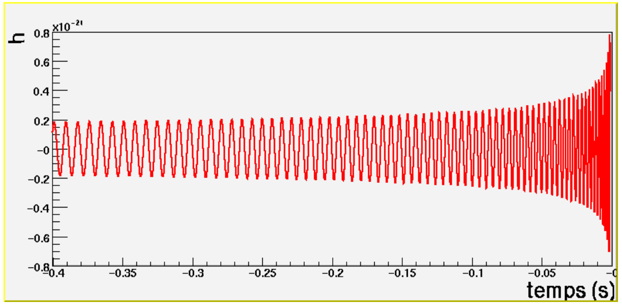

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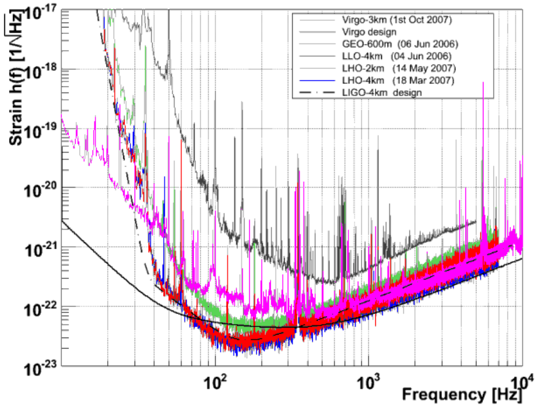
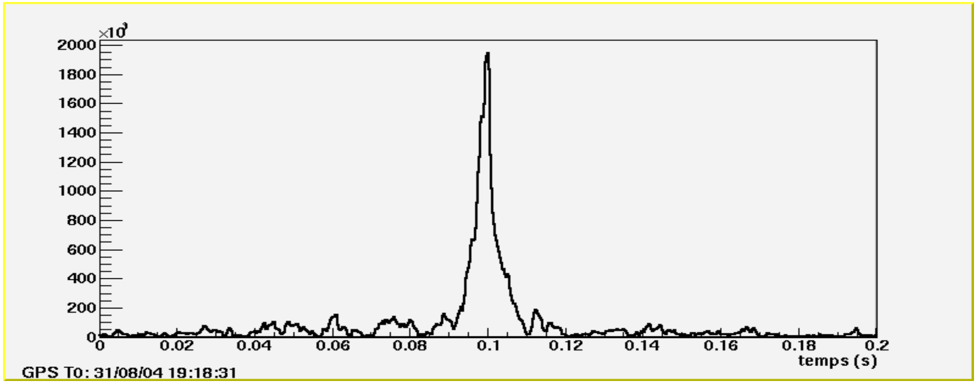




Matched filtering in practice (II)

- The phase of the chirp signal is unknown

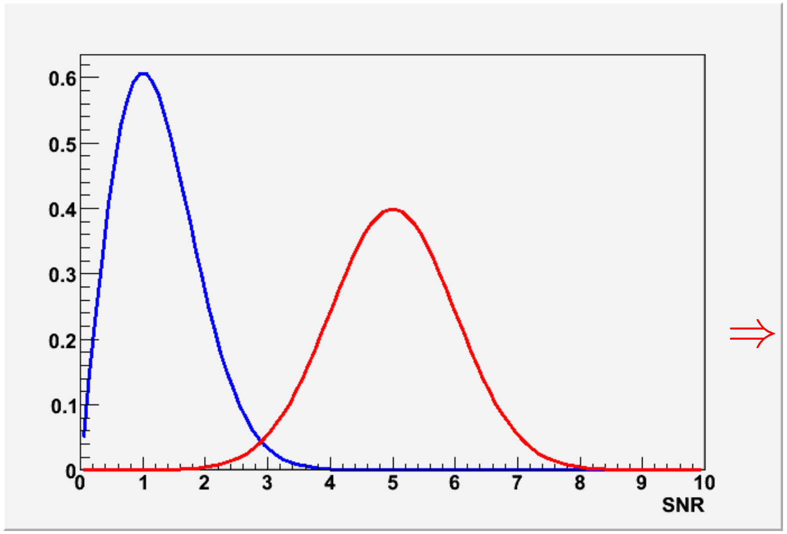
$$h(t) = A\,[h_c(t)\cos\Phi + h_s(t)\sin\Phi]$$

cosine and sine phases of the waveform

» The SNR has to be maximized over all possible values of Φ

Filter with T_{0° and T_{90° and take quadratic sum

$$S^2 = \sqrt{{S_{0^\circ}}^2 + {S_{90^\circ}}^2}$$



Noise has a χ^2 distribution with 2 degrees of freedom $p(\rho) = \rho e^{-\rho^2/2}$

Signal has a non-central χ^2 distribution

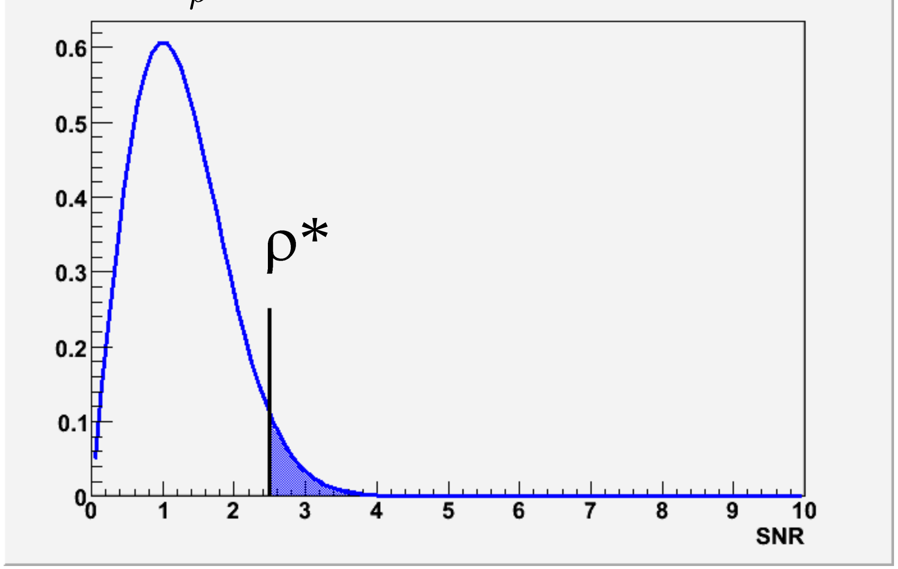
$\Rightarrow$ Gaussian distribution if signal strong enough

Matched filtering is optimal

- If the noise is Gaussian, the matched filtering provides the optimal statistic
  - » Selecting events by setting a threshold on the SNR $\rho > \rho^*$ guarantees the lowest false alarm probability for a given detection probability

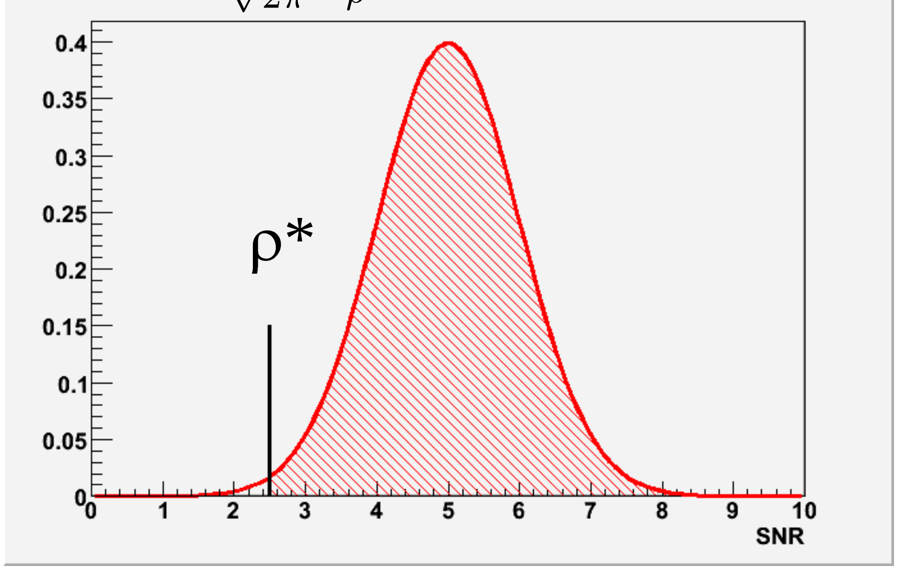
False alarm probability

$$\int_{\rho^\star}^{\infty} \rho e^{-\rho^2/2} d\rho = e^{-\rho^{\star 2}/2}$$



Detection probability for signal with SNR ρ_S

$$\frac{1}{\sqrt{2\pi}} \int_{\rho^\star}^{\infty} e^{-(\rho-\rho_S)^2/2} d\rho$$



Scanning the parameter space (I)

- The template T(t) depends on the source parameters: masses, spin
  - It is not possible to maximize analytically the SNR for those intrinsic parameters
  - It is necessary to try a family of templates sampling the parameter space
  - Let us forget about spin and concentrate on the mass parameters
- Now that we know the optimal filter, let us redefine the inner product as:

$$(a,b) = 4\,\Re \int_0^\infty \frac{\tilde{a}(f)\tilde{b}^\star(f)}{S_h(f)}\,df$$

With properly normalized templates, the filtered SNR for template u is $SNR = (h,u)$, or rather $\max_{\phi_c,t_c}(h, u(\theta)e^{i(2\pi f t_c+\phi_c)})$

$$\lambda = (\phi_c, t_c, \theta) \text{ with } \theta \text{ the intrinsic parameters}$$

If data containing signal $u(\boldsymbol{\lambda_1})$ is filtered with a template with different parameters $u(\boldsymbol{\lambda_2})$ the fraction of the optimal SNR recovered is given by the ambiguity function $A(\lambda_1,\lambda_2) = (u(\lambda_1), u(\lambda_2))$ maximized over extrinsic parameters, i.e. is given by the match:

$$M(\theta_1,\theta_2) = \max_{\phi_c,t_c}(u(\theta_1), u(\theta_2)e^{i(2\pi f t_c+\phi_c)})$$

Scanning the parameter space (II)

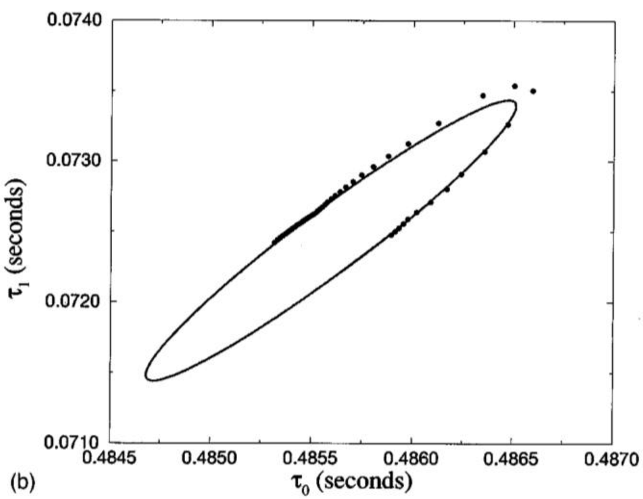
- From the match, define a metric on the parameter space

$$g_{ij}(\theta) = -\frac{1}{2}\left.\frac{\partial^2 M(\theta,\Theta)}{\partial\Theta^i\partial\Theta^j}\right|_{\Theta=\theta}$$

- In the regime 1-M << 1 the match can be approximated by

$$M(\theta, \theta+\delta\theta) \sim 1 - g_{ij}\delta\theta^i\delta\theta^j$$

[Ref.8]



- Instead of the masses m_1, m_2, it is more convenient to use as parameters:

$$\tau_0 = \frac{5}{256} M^{-5/3}(\pi f_0)^{-8/3}\eta^{-1}$$

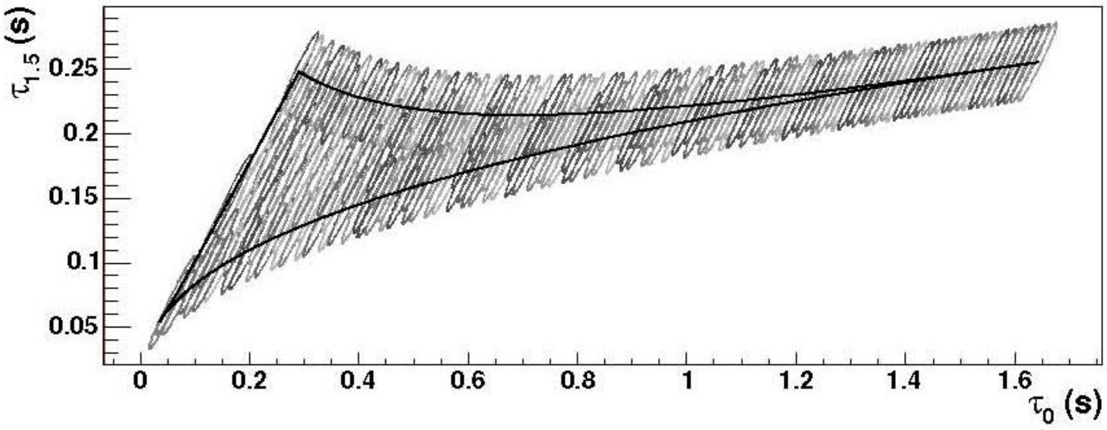
$$\tau_1 = \frac{5}{192} M^{-1}(\pi f_0)^{-2}\left(\frac{743}{336\eta} + \frac{11}{4}\right)$$

- For matches above ~95%, isomatch contours are ellipses
- In the τ_0, τ_1 space, the metric components g_{ij} are constant at 1PN order, and have small variations at higher order.

Scanning the parameter space (III)

- Each isomatch contour defines a region of the parameter space which overlaps with the template in the center with a match better than some value M
- The template in the center can be used to search for signals in that region of the parameter space, at the price of a controlled loss of SNR (< 1 – M)
- Templates should be placed over the parameter space in order to
  - Achieve coverage of space (no « holes »)
  - Preserve search efficiency: keep number of templates as low as possible

[Ref.9]



- Things become difficult when other parameters (like spins) need to be taken into account (> 2 dimensions)

- To be efficient and safe
  - Take into account variations of ellipse size and orientation across the parameter space
- For Virgo at design sensitivity, to search the 1-30 $M_\odot$ space with f_0 = 30 Hz and a minimal match of 98%, ~ 50000 templates needed

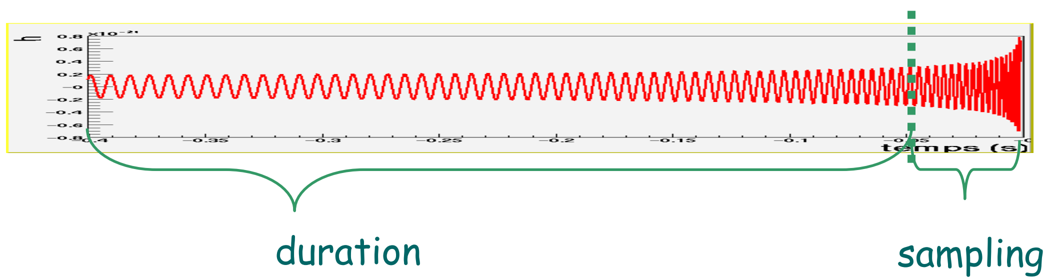
Hierarchical methods

- Template based searches can be computationally demanding when the number of templates is large
  - » Depends on the detector bandwidth
  - » Depends on the number of parameters to be scanned
- Hierarchical methods aim at reducing the computing needs
  - » Conduct search in several steps, e.g.:
    - 1st step: use a coarser template bank, i.e. a smaller minimal match
      lower threshold to compensate for reduced signal-template match and keep good detection probability ($\Rightarrow$ increased FAR)
    - 2nd step: for triggers above threshold at 1st step, refine analysis with a higher density template bank
    - Computing gain can be of order ~ 25 [Ref.10]
      - Depends on background!

Hierarchical methods: the multi-band approach (I)

- The computing cost of a matched filter search based on a template bank is due to
  - » The number of templates ⇐ detector bandwidth
  - » The size of the FFT involved in the matched filtering operation
    - – Template duration ⇐ dominated by the low frequency evolution
    - – Sampling frequency ⇐ imposed by the high frequency content of the signal

- The analysis can be split in a few bands (two or three)



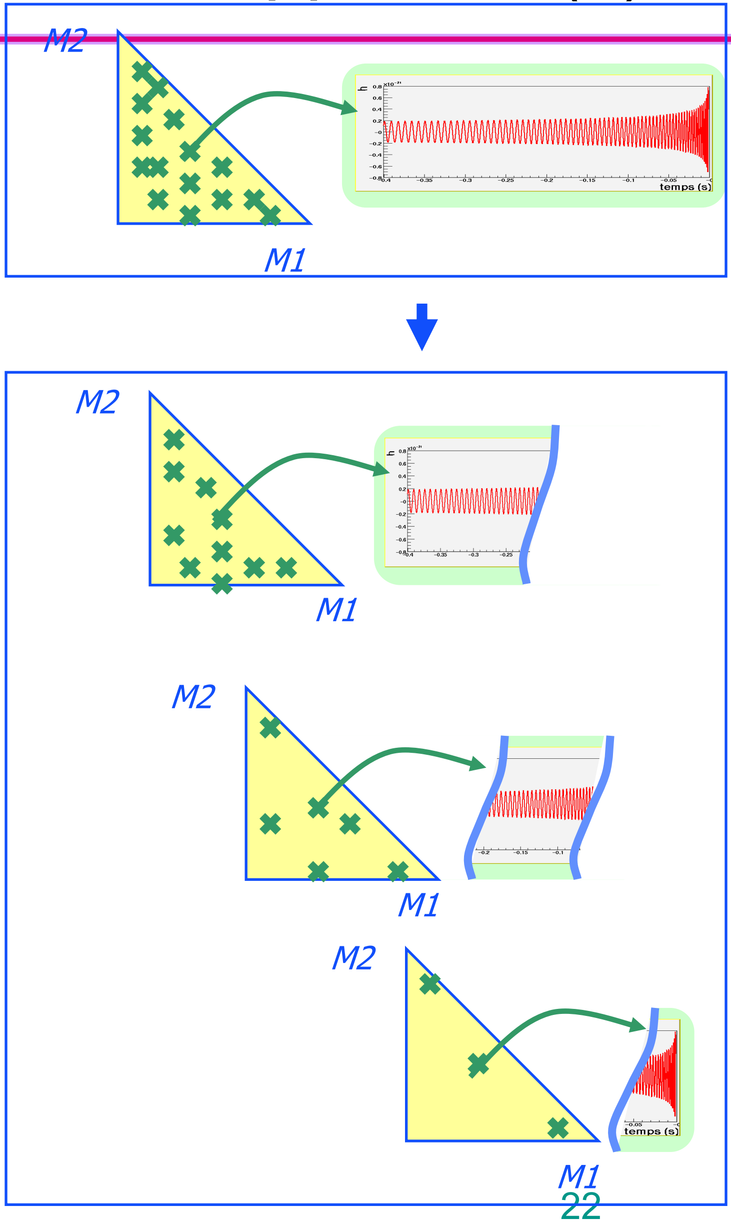
[Ref.11]

$$\int_{f_{min}}^{f_{max}} \tilde{h}(f)\tilde{Q}^*(f)df = \int_{f_{min}}^{f_1} \tilde{h}(f)\tilde{Q}^*(f)df + \int_{f_1}^{f_2} \tilde{h}(f)\tilde{Q}^*(f)df + \int_{f_2}^{f_{max}} \tilde{h}(f)\tilde{Q}^*(f)df$$

Hierarchical methods: the multi-band approach (II)

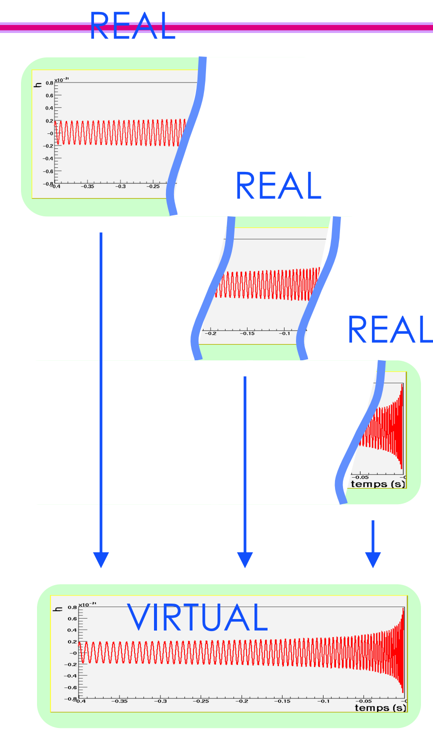
- Build one bank of *real* templates per frequency band
  - » Less templates in each bank
  - » Short templates in high frequency band
  - » Data can be downsampled for the low frequency bands filtering
  - ⇒ Less and shorter FFTs

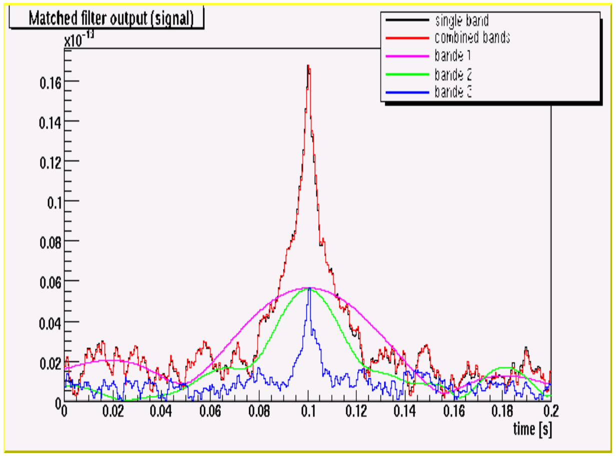
- Filter data with each template bank
  - » Complex filtered signal (phase and quadrature) for each template



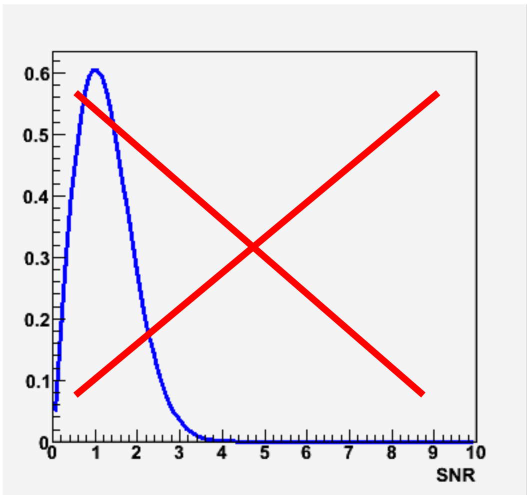
Hierarchical methods: the multi-band approach (III)

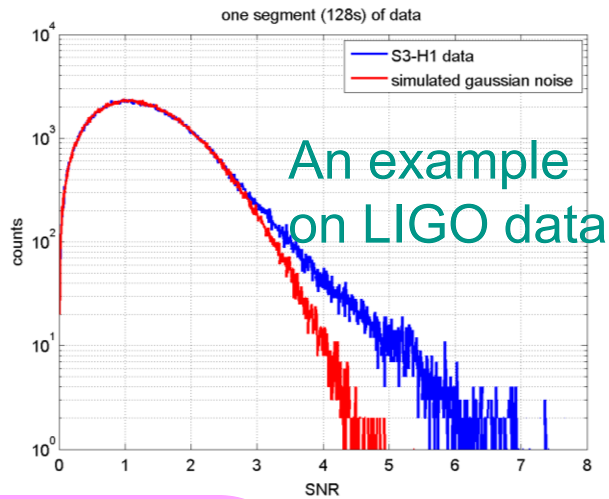
- Build a bank of *virtual* templates on the full frequency band
  - » To each virtual template associate a real template in each frequency band
- Add coherently the filtered signals
  - » Interpolate low frequency band results
  - » Apply time delays and phase offsets between frequency bands
    - – Take signal evolution into account
  - » Conditional combination
    - – If SNR exceeds some threshold in at least one of the bands
    - – Built-in hierarchy
- Final threshold is applied on combined signal

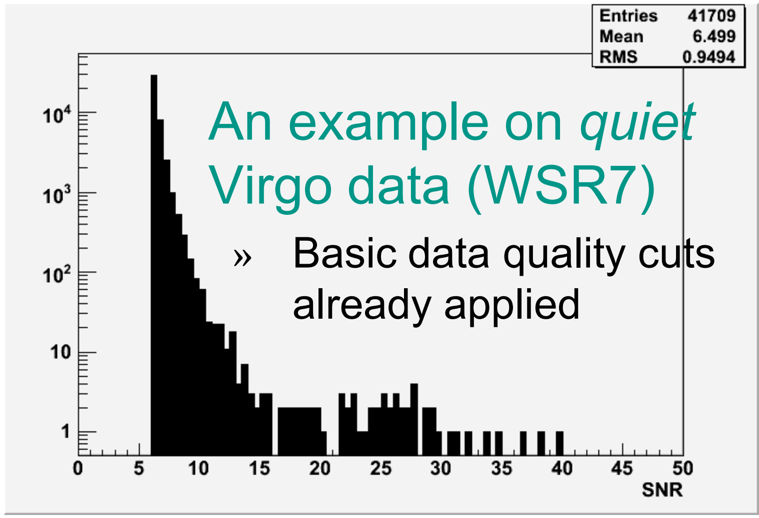




Background is not Gaussian!







- Coincidences
  - Reduce false alarm rate by requiring coincident triggers in several detectors
  - Allows to estimate the non-Gaussian background from the data themselves
- Instrumental vetoes
  - Check for anomalies in detector behavior, statistically associated with excess triggers
- Signal based vetoes
  - Check trigger internal consistency with expected CB signal
- Special case of targeted searches
  - e.g. GRB
  - Estimate background “off source”

Coincidences

- Require coincident triggers in 2 or more detectors
  - » Check parameter consistency within allowed « windows »

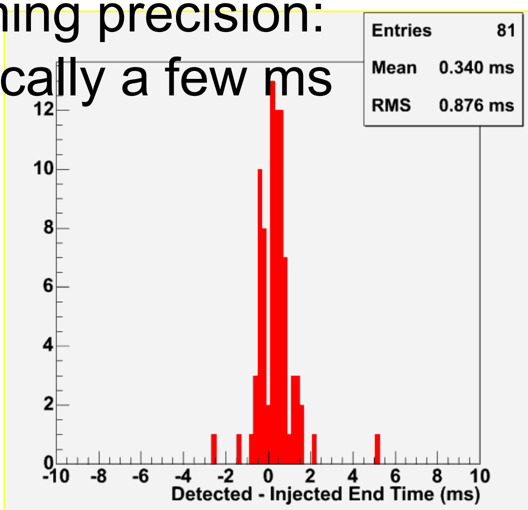
    $$\Delta t, \ \Delta\mathcal{M}, \Delta\eta$$

  - » Smaller coincidence windows $\Rightarrow$ larger reduction of FAR
    - – Window size depends on the resolution with which each detector is able to determine those parameters
    - – Δt must allow for time of flight between detectors

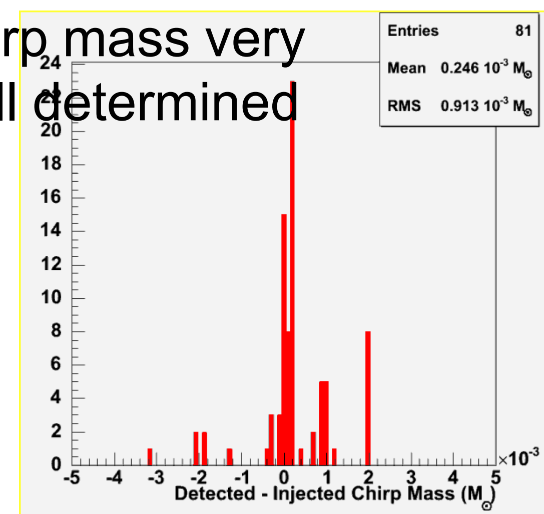
      LIGO Hanford – LIGO Livingston: 10 ms

      Virgo – LIGO: 30 ms

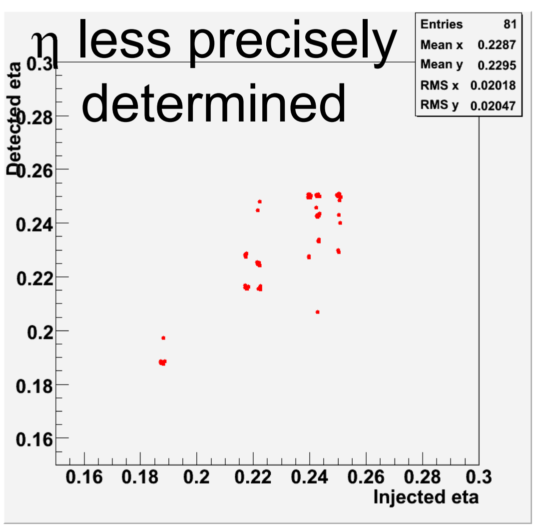
Timing precision: typically a few ms



Chirp mass very well determined



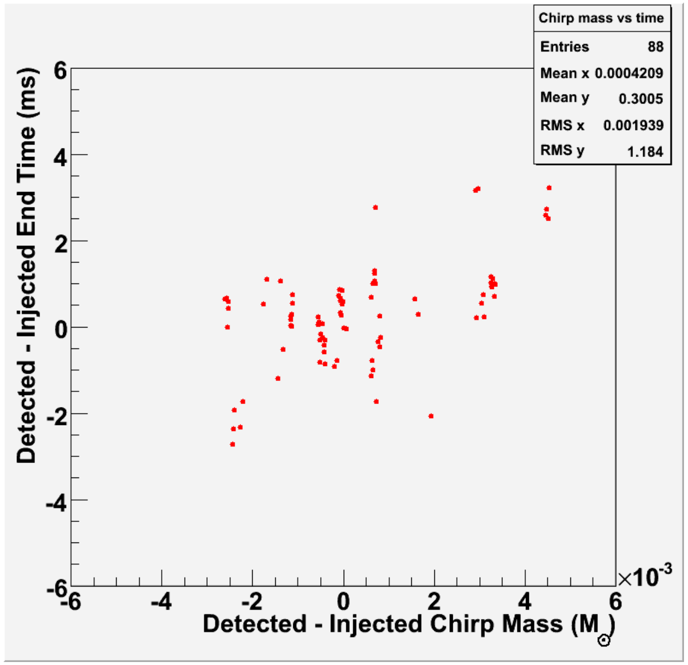
η less precisely determined



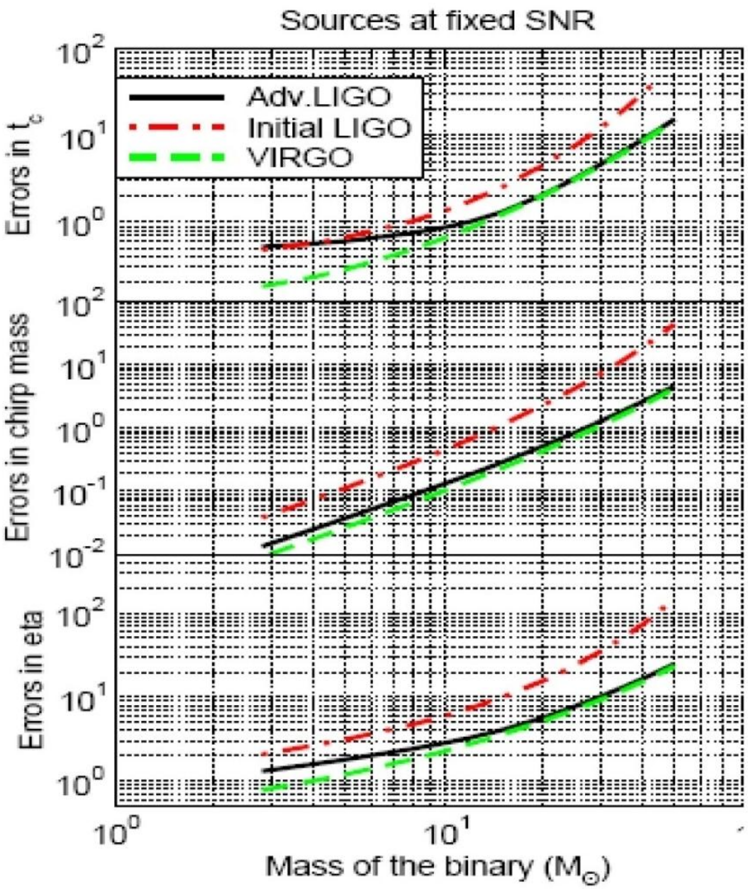
Coincidences (II)

- Fixed coincidence windows are not optimal

Parameters are correlated

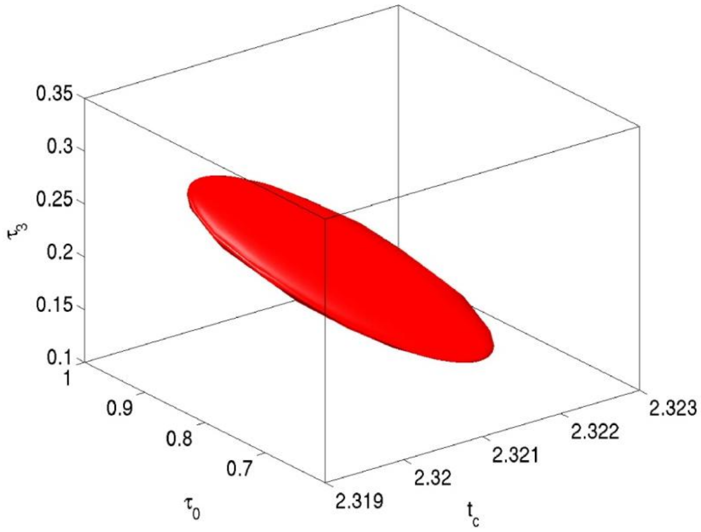


Errors on parameters vary across the parameter space



[Ref.12]

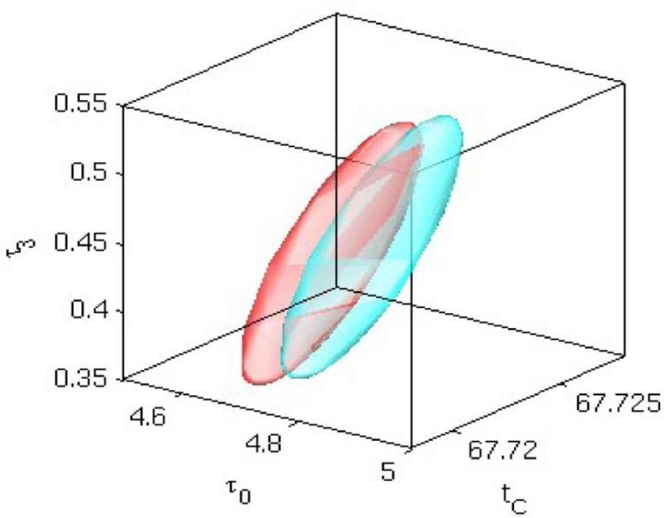
Coincidences (III)



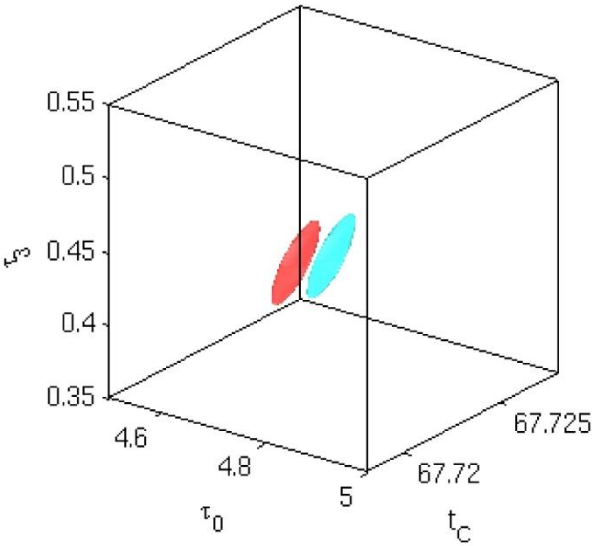
- Use ellipsoids to define coincidences
  - » Builds in correlation and accuracy variation
- Achieves background reduction of a factor 10

[Ref.13]

Coincident

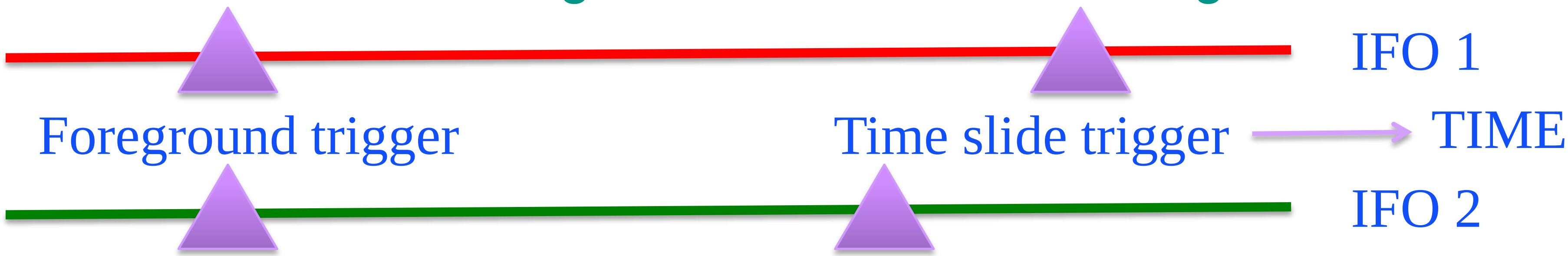


Not coincident



Coincidences (IV)

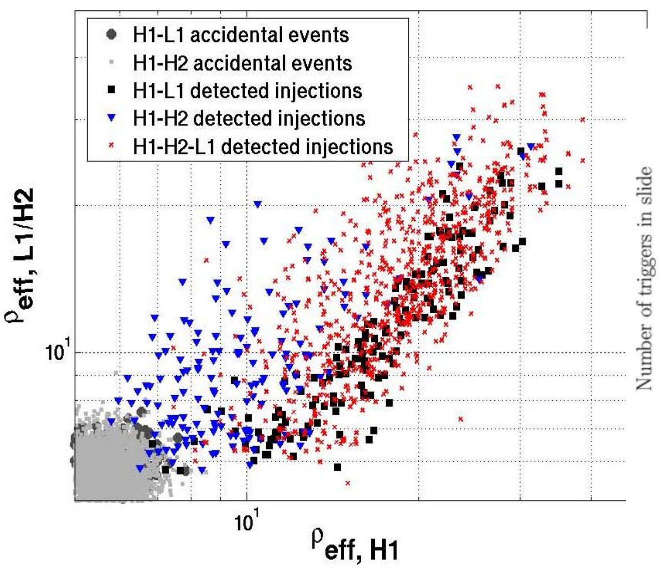
- Allow to estimate background from the data using time shifts

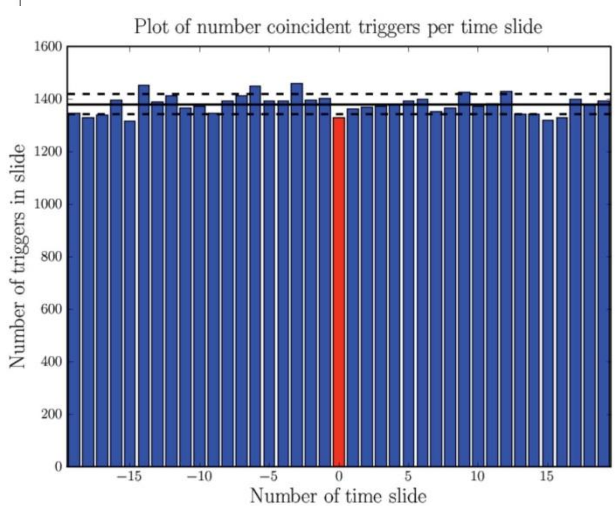


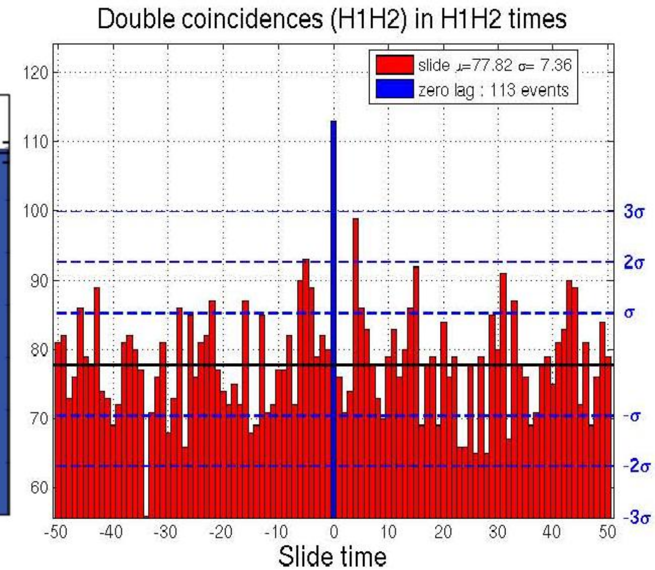
  » Works well for distant sites

  » Co-located detectors (LIGO H1-H2) usually show excess coincident background with respect to time slides estimates [Ref.14]

    – Evidence for correlated noise



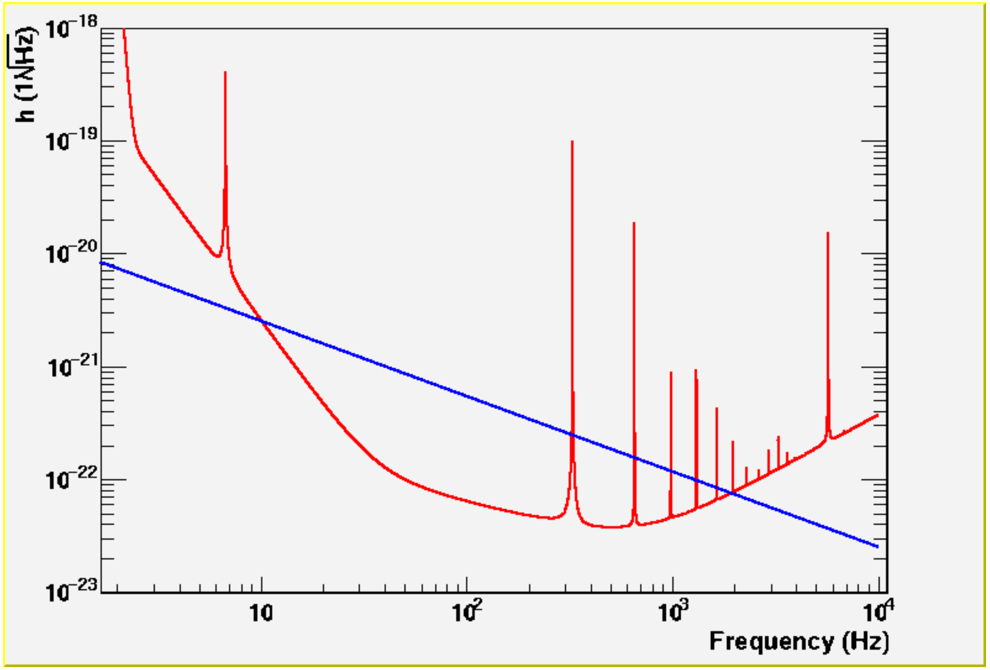


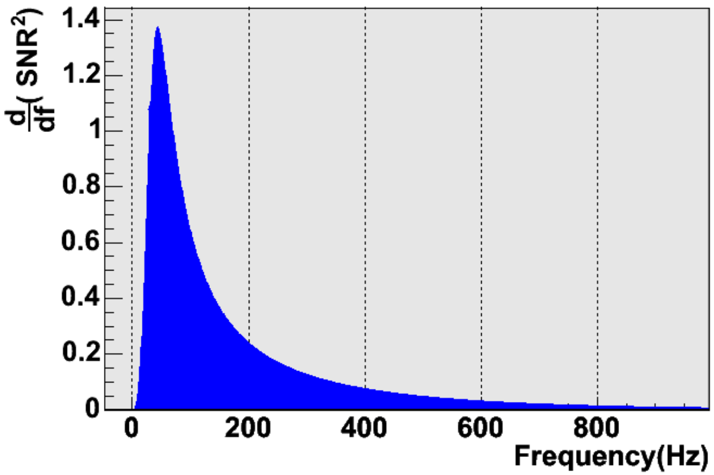


Signal based vetoes: χ^2 test (I)

- Basic idea: look at how the SNR is distributed across the detector bandwidth and check whether this is consistent with what is expected from a true signal

$$\mathrm{SNR}^2 = 4\int_0^\infty \frac{|\tilde{h}_S(f)|^2}{S_h(f)}\,df \quad \sim A\int_0^{f_{max}} \frac{f^{-7/3}}{S_h(f)}\,df$$





Signal based vetoes: χ^2 test (II)

- The matched filter integral can be written as a sum over distinct frequency bands

$$(a,b) = \sum_{j=1}^{p} (a,b)_j \quad \text{with} \quad (a,b)_j = 4\,\Re \int_{\Delta f_j} \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_h(f)}\,df$$

- The frequency intervals are chosen so that for a true signal the SNR is uniformly shared among the frequency bands

$$\Delta f_j \text{ such that } (T,T)_j = \frac{1}{p}(T,T) \quad \text{or} \quad \int_{\Delta f_j} \frac{f^{-7/3}}{S_h(f)}\,df = \frac{1}{p}\int_0^{f_{max}} \frac{f^{-7/3}}{S_h(f)}\,df$$

- The filtered SNR can be written as

$$\rho = \sum_{j=0}^{p} \rho_j \text{ with } \rho_j = (h,Q)_j$$

- A discriminating statistics is built

[Ref.15]

$$\chi^2 = p\sum_{j=0}^{p} (\Delta\rho_j)^2 \text{ with } \Delta\rho_j = \rho_j - \frac{\rho}{p}$$

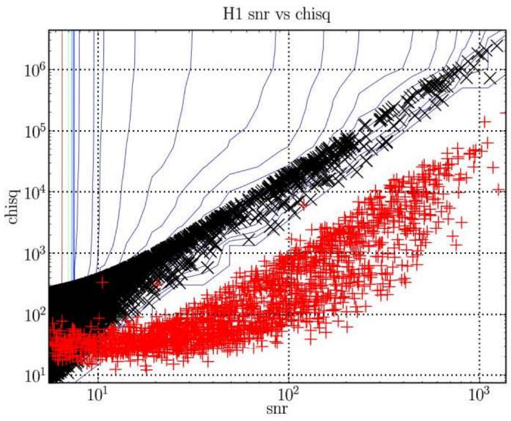
- If the noise is stationary and Gaussian, the χ^2 has a χ^2-distribution with p-1 degrees of freedom both for noise and for true signals
- Excess noise is expected to produce χ^2 values which are outliers with respect to the Gaussian noise/signal distribution

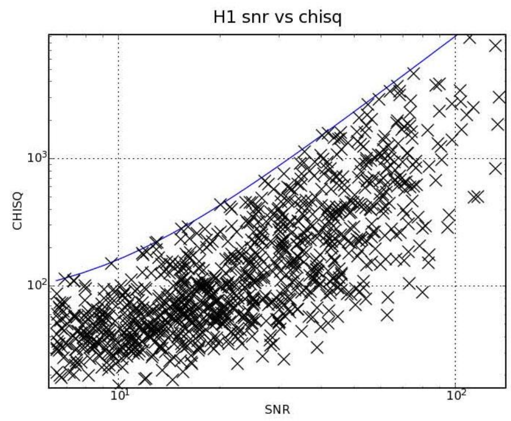
Signal based vetoes: χ^2 test (III)

- χ^2 distribution for true signals in practice
  - Large SNR events tend to show larger χ^2 values than expected from the naive distribution
  - An effect of using template banks
  - The slight mismatch between the signal and the template is enough to evidence differences between the expected SNR frequency distribution and the measured one $\Rightarrow$ high χ^2
  - The cut used to eliminate background must allow some quadratic dependence of the χ^2 on the SNR
  - Apply threshold on variable

$$\xi^2 = \frac{\chi^2}{p(1+\delta^2\rho^2)}$$

- Tuning
  - Adjust p, δ and threshold not to reject true signals
  - Cut must be loose enough to be robust with respect to missing features in the templates
    - Spin
    - Ringdown

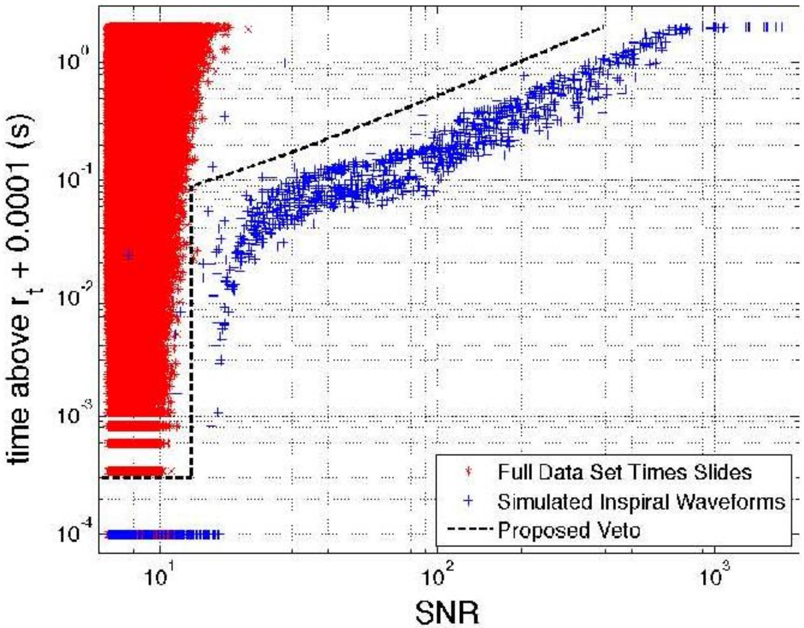


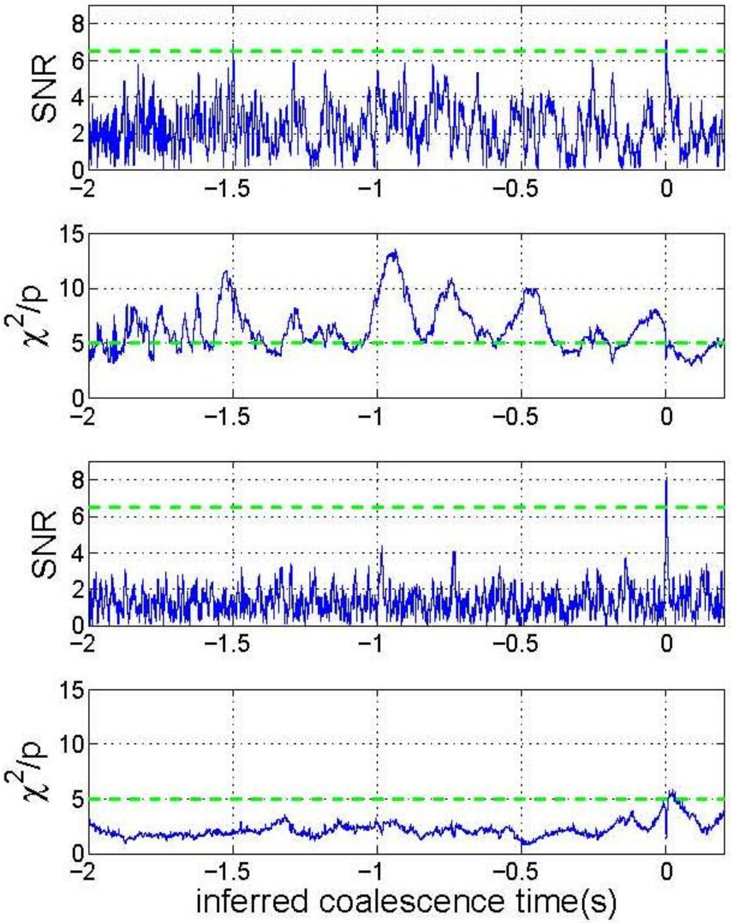


[Ref.16]

Signal based vetoes: $\chi^2(t)$

- Look at $\chi^2(t)$: "r^2 veto"
  - » Use as discriminating variable the time spent by $\chi^2(t)$ above some threshold in some time window prior to the measured coalescence time





[Ref.16]

Signal based vetoes: drawbacks

- Signal based vetoes are powerful but
  - They are usually computationally expensive
  - They do not provide any feedback on the detector
  - They cannot be applied when phenomenological detection templates are used

Instrumental vetoes (I)

- Identify anomalies in the detector behavior/environment statistically coincident with CB triggers
  - » Ideally, understand origin of bad behavior and fix it
  - » Help clean up the background by eliminating the corresponding triggers
- Instrumental vetoes should be
  - » Efficient: eliminate false triggers, especially triggers with high SNR
  - » Relevant: they should be often enough associated with triggers (use percentage)
  - » Cheap: they should not eliminate a large fraction of the data (dead time)
  - » Safe: they should not eliminate true signals
    - – safety checked with hardware injections ☚

☛Hardware injections are simulated signals physically in the interferometer by acting on the mirrors, to check the analysis pipeline as a whole, from the reconstruction of the h(t) signal to the trigger production, and to check the safety of vetoes.

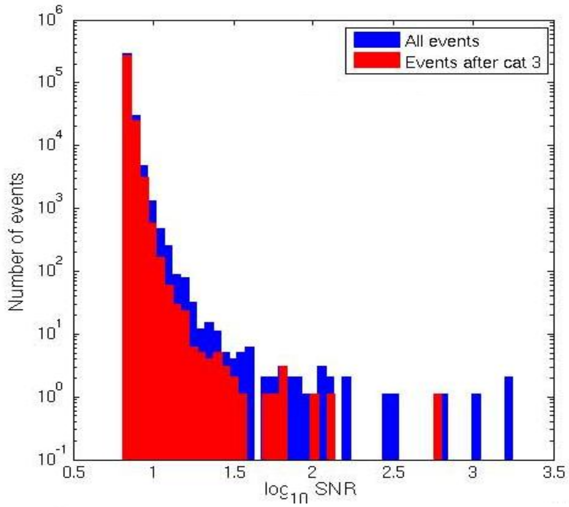
Instrumental vetoes (II)

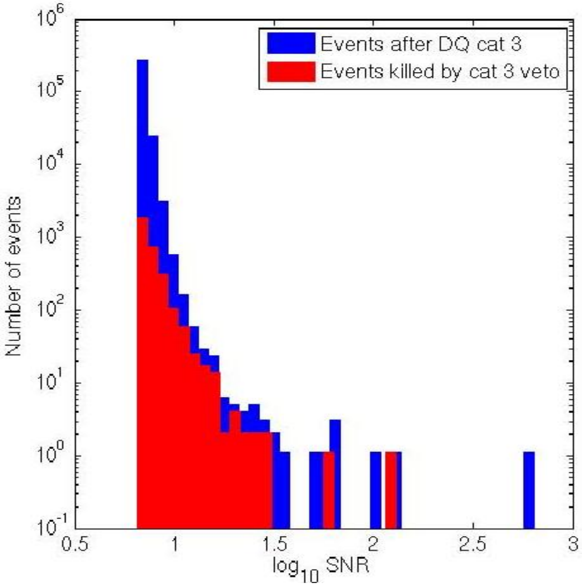
- Vetoes are categorized according to severity, statistical correlation and dead time
  - » Category 1
    - – Data not suitable for being analyzed
    - – e.g.: detector not at operating point; missing data…
  - » Category 2
    - – Well understood instrumental problems
    - – Strong statistical correlation
    - – Usually low dead time
    - – e.g.: overflow in ADC digitizing photodiode signals
  - » Category 3
    - – Suspected instrumental problems
    - – Positive statistical correlation, but not well understood
    - – Dead time can be large
    - – This category also includes ad-hoc vetoes based on auxiliary channels
    - – e.g.: high seismic activity, strong wind
  - » Category 4
    - – Poorly understood, weak but positive correlation
    - – May veto whole noisy epochs

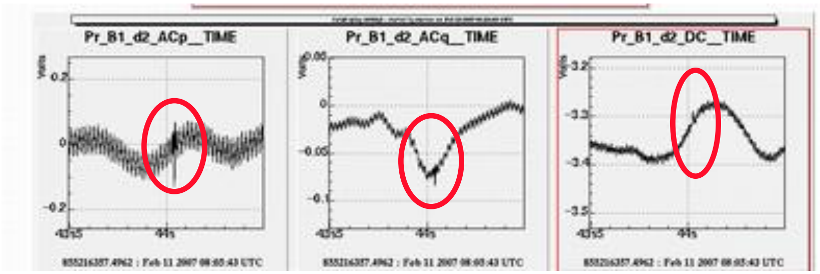
Instrumental vetoes (III)

- Vetoes based on data quality flags pointing out understood detector / environment bad behavior

- Vetoes based on auxiliary channels showing glitch correlation with GW channel
- Ad-hoc vetoes
  - » Use photodiode signals to veto triggers caused by dust particles passing through beam
    - – Keep veto safe!

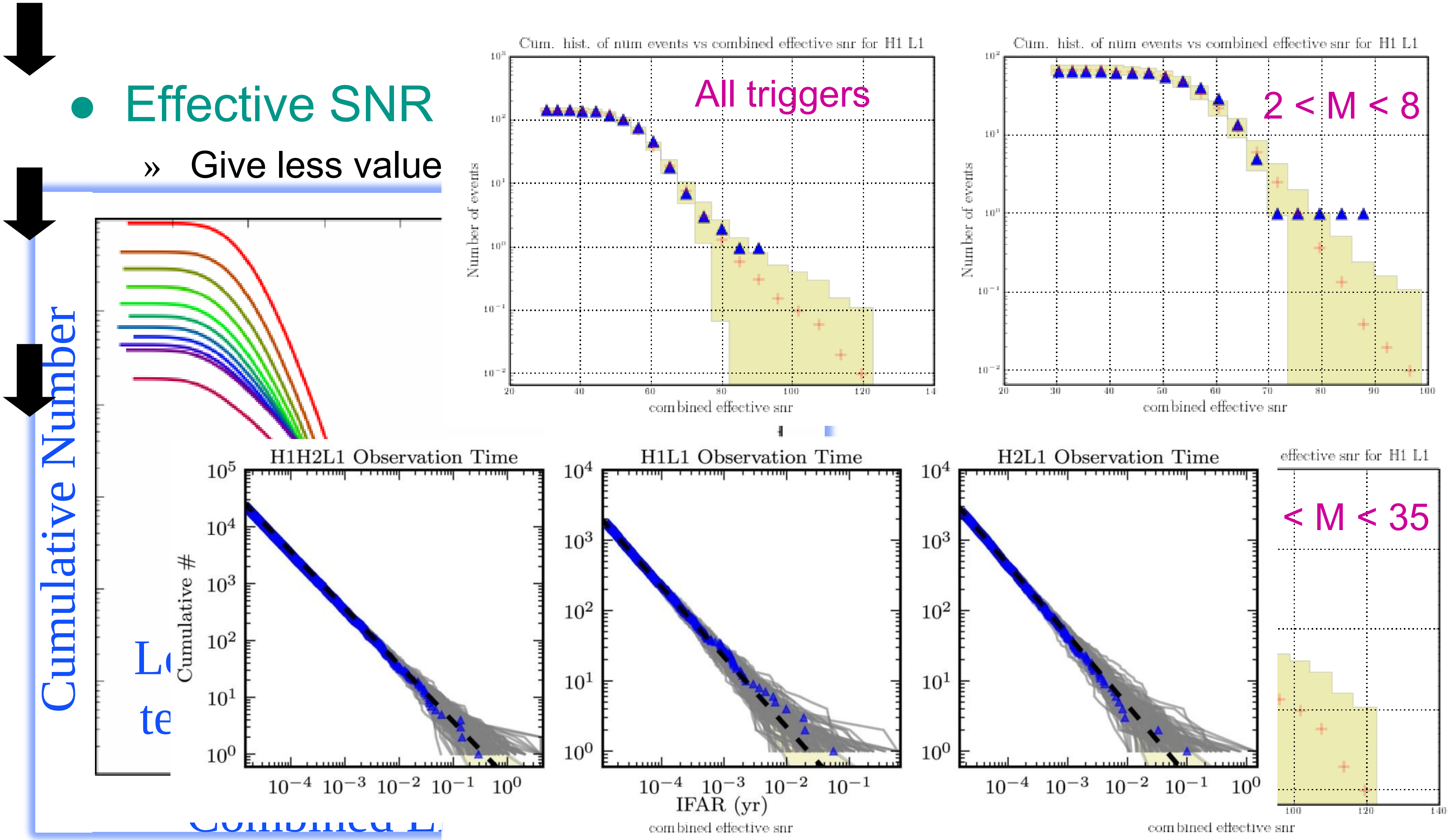




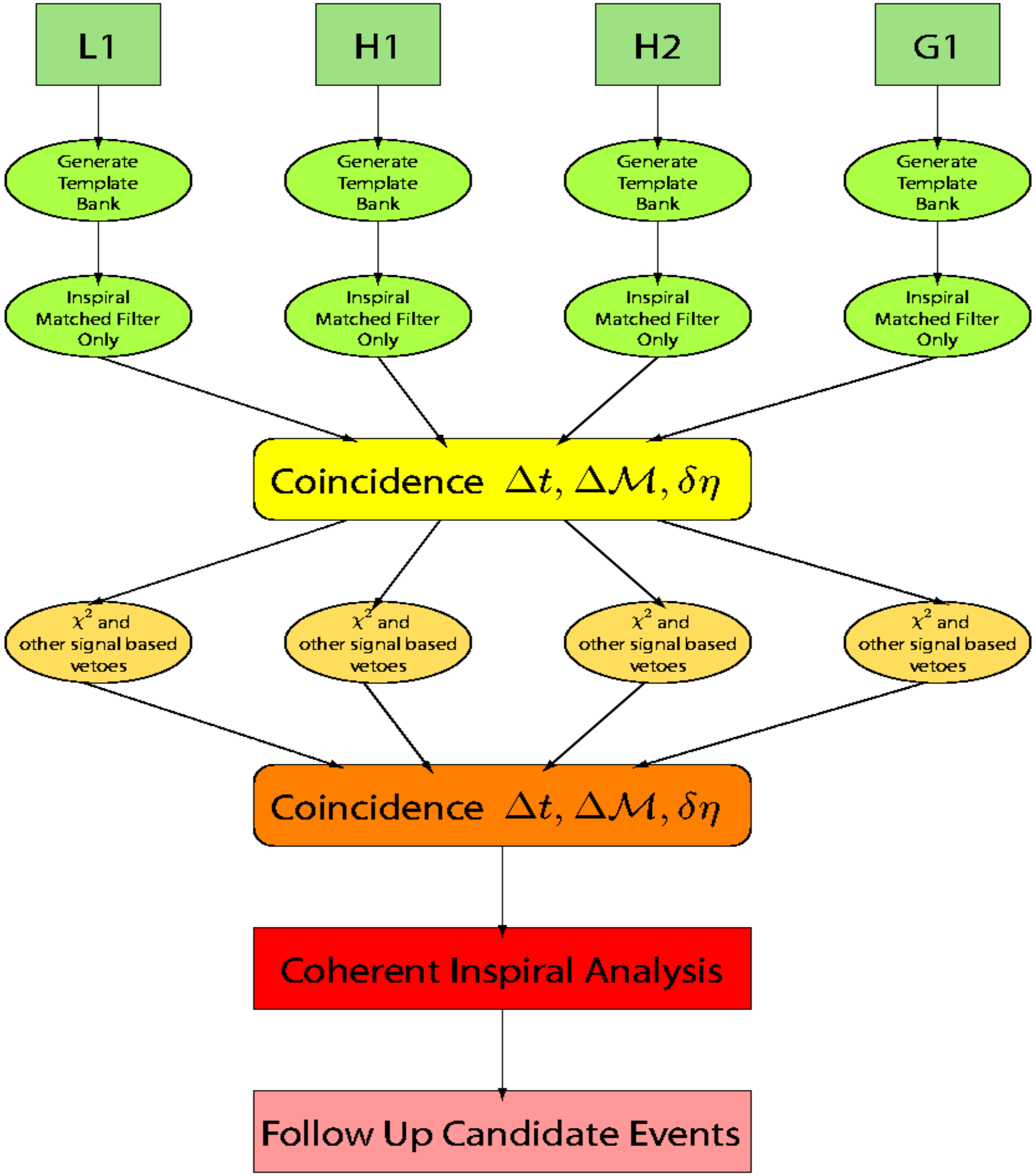


Improving the detection statistic

- SNR
- Effective SNR
  - » Give less value



Combining all this: the LIGO pipeline



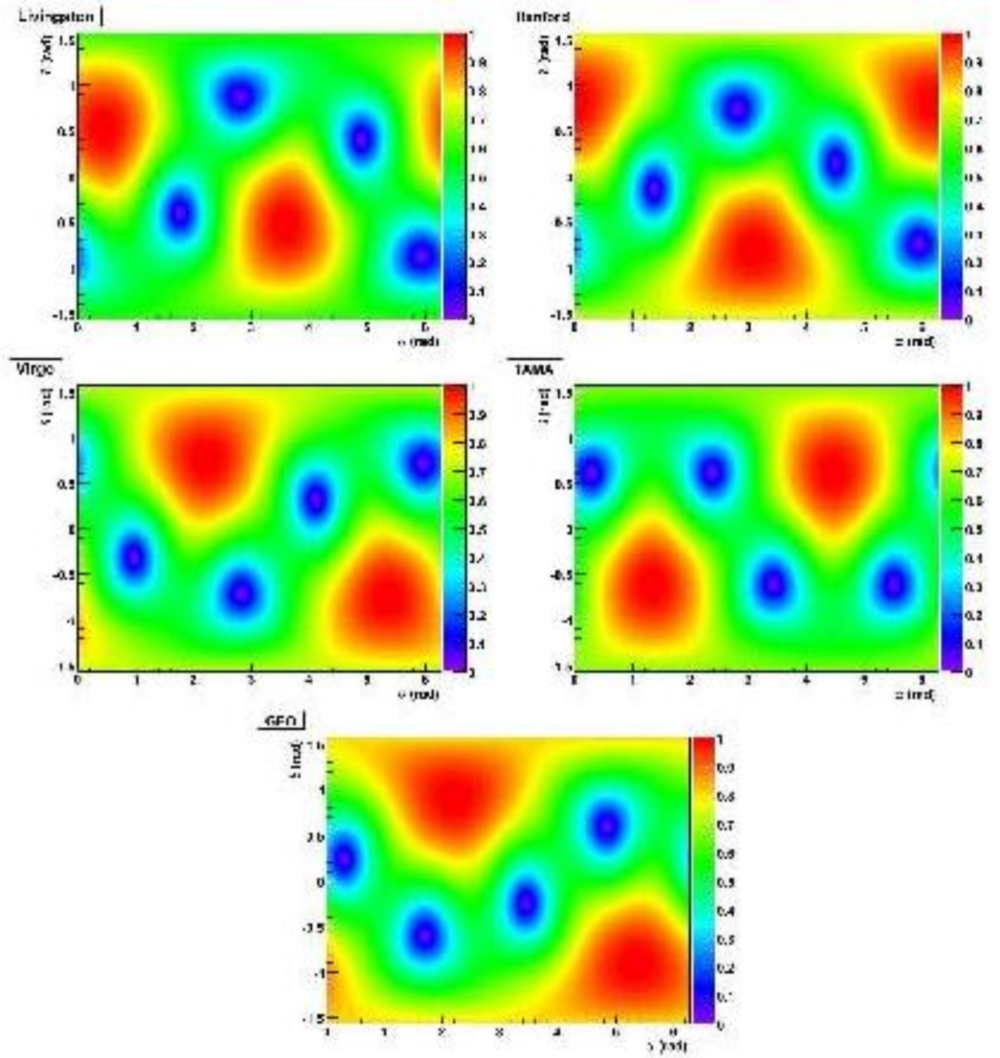
[Ref.18]

Network analysis (I)

- Several detectors are a tool to cope with excess noise, but they also allow to extract more science
- With a single interferometer
  - Can in principle measure masses, spins, and effective distance to compact binary coalescence.
- With two geographically separated interferometers
  - Can in principle locate source on sky annulus via time delay
  - Can in principle also measure inclination, polarization angle as function of sky location
- With three geographically separated interferometers
  - Can in principle measure the sky position and all other parameters of the binary

Network analysis (II)

- Differently located and oriented detectors have different sensitivities for a given source direction
  - » e.g.: sensitivity for a circularly polarized GW



Coherent analysis (I)

- If the noise is stationary and Gaussian, the optimal strategy is to perform a coherent search
  - » Treat each detector signal as a component of a global detector signal, and perform matched filter with global templates

Correlators at detector i for the two templates in quadrature are $C_0^i(t)$ and $C_{\frac{\pi}{2}}^i(t)$

Time delay $\tau_i(\theta,\phi)$ depending on source direction

$$\mathrm{SNR}^2_{\mathrm{network}} = \sum_{i,j} \; p_{i,j}(\theta,\phi) \left[C_0^i(t-\tau_i(\theta,\phi)) \; C_0^j(t-\tau_j(\theta,\phi)) + (0 \rightarrow \tfrac{\pi}{2}) \right]$$

Weighting matrix, depending on the location/orientation of the detectors, and on their relative sensitivity

$$\mathrm{SNR}^2 = \kappa^2 \left\{ \tfrac{1}{2} \sum_i |E_i(\theta,\phi,\iota,\psi)|^2 \right\}$$

Intrinsic source strength

Extended beam patterns, depending on detector location, orientation, sensitivity

Coherent analysis (II)

- Performing network matched filtering is computationally expensive
  - » A larger parameter space should be scanned [Ref.19]
    - Arrival time, source mass parameters, source direction…
- Non-Gaussian noise prevents from relying only on a coherent search anyway
- Used at follow-up level

Setting upper limits

- In the absence of a detection, set upper limits on the coalescence rate
  - Use loudest event statistic in a Bayesian approach [Ref.22]
  - Probability that all signal events have SNR below some value ρ:

$$P(\rho|\mu) = e^{-\mu\epsilon(\rho)} \text{ (signal Poisson distributed)}$$

$$\mu = R\,T \text{ with R the event rate and T the observation time}$$

$$\epsilon(\rho) \text{ : signal detection efficiency with SNR threshold } \rho$$

  - Neglecting the background, posterior probability distribution for μ:

$$P(\mu < \mu_p|\rho_{max}) = \mathcal{N}^{-1} \int_0^{\mu_p} d\mu \; p(\mu) \; p(\rho_{max}|\mu) \text{ with } p(\rho|\mu) = dP(\rho|\mu)/d\rho$$

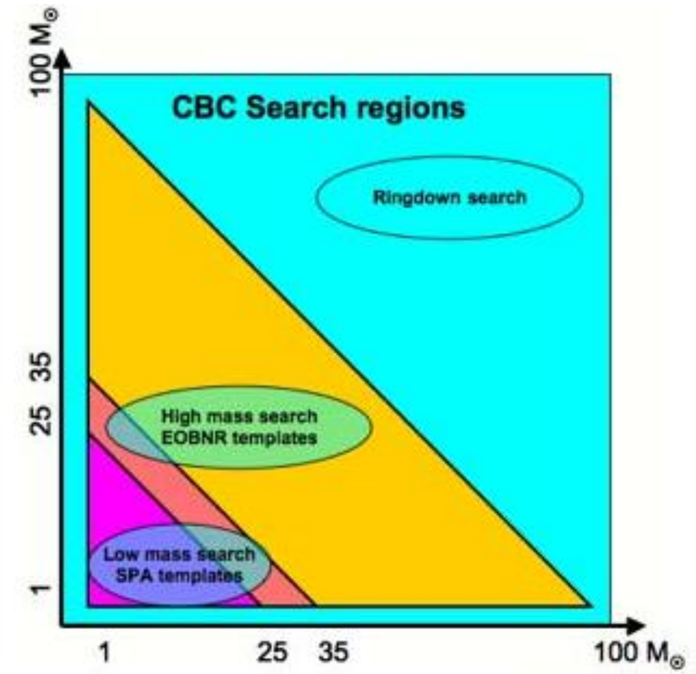
$$\text{Solve } \; p = P(\mu < \mu_p|\rho_{max}) \text{ for } \mu_p \text{ to get 100p\% CL upper limit}$$

$$\text{With uniform prior } p(\mu) \; R_{90\%} = \frac{3.890}{T\epsilon(\rho_{max})}$$

  - The background can be taken into account to get better upper limit
- Best current upper limit for BNS (LIGO S5)
  - $R_{90\%} = 1.4\ 10^{-2}\ yr^{-1}\ L_{10}^{-1}$ [Ref.16]

Low mass search (I)

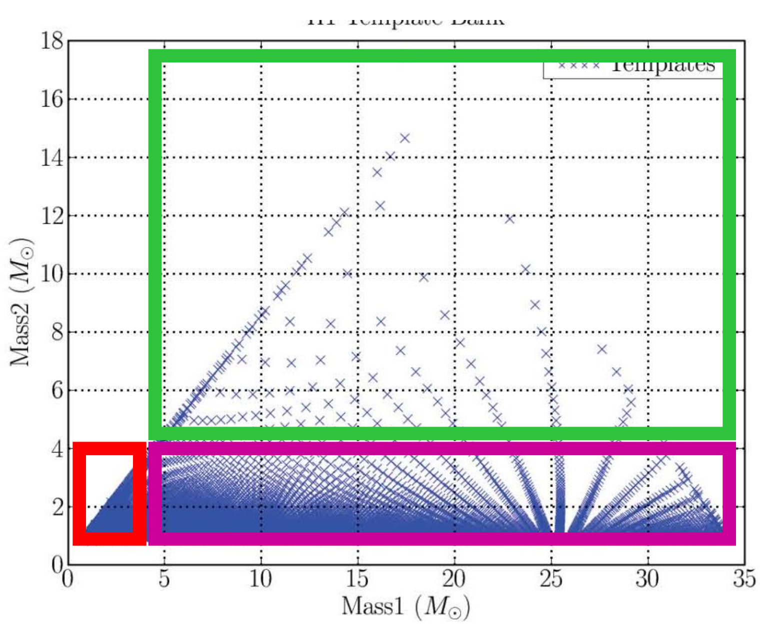
- Search for binary systems consisting of neutron stars and/or black holes, with total mass between 2-35 $M_\odot$ and a minimum component mass of 1 $M_\odot$
- 2nd order post-Newtonian templates



BNS

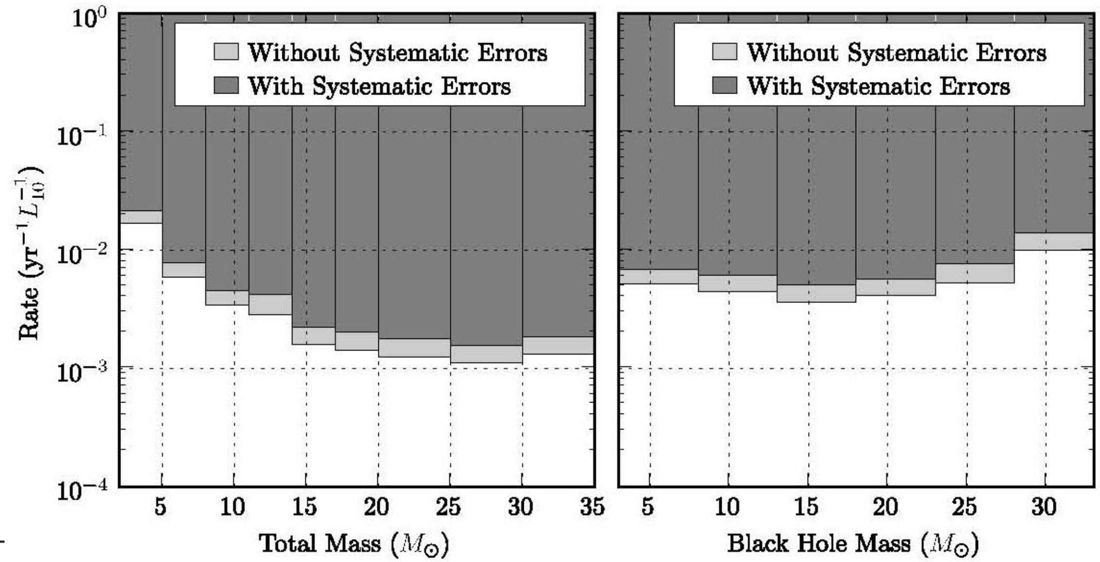
BBH

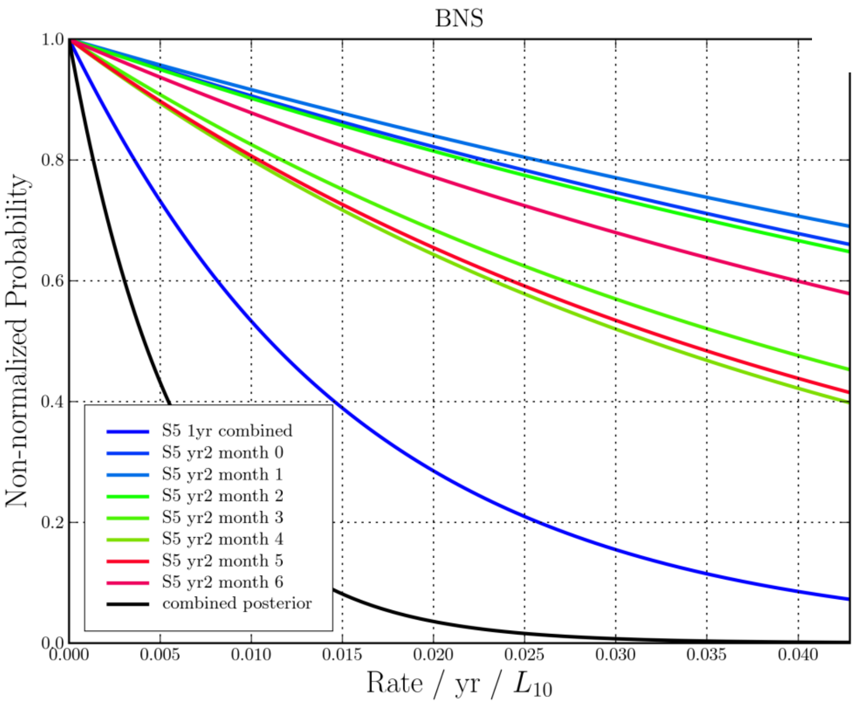
NSBH



Low mass search (II)

- Mass dependent upper limits can be derived





- Results from successive searches can be combined
  - S5 12-18 months combined with S5 first year
  - UL a factor 3 lower than S5 1st year
    - Less data, but more sensitive

Low mass search (III)

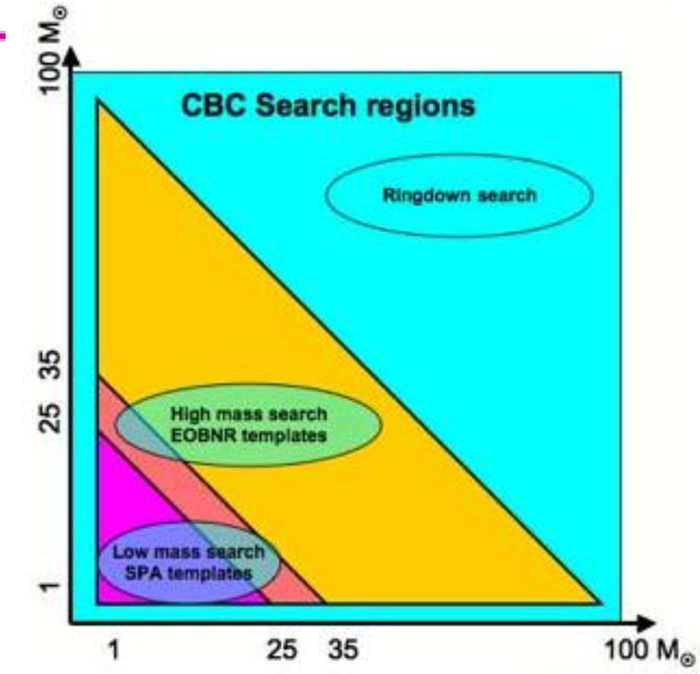
- Compare rate upper limits to astrophysical expected rates
- Best current results
  - » BNS rate 90% confidence = $1.4 \times 10^{-2}\, L_{10}^{-1}\, yr^{-1}$
  - » BBH rate 90% confidence = $7.3 \times 10^{-4}\, L_{10}^{-1}\, yr^{-1}$ [Ref.16]
  - » NSBH rate 90% confidence = $3.6 \times 10^{-3}\, L_{10}^{-1}\, yr^{-1}$
- Astrophysical optimistic rates
  - » BNS rate = $5 \times 10^{-4}\, L_{10}^{-1}\, yr^{-1}$
  - » BBH rate = $6 \times 10^{-5}\, L_{10}^{-1}\, yr^{-1}$
  - » NSBH rate = $6 \times 10^{-5}\, L_{10}^{-1}\, yr^{-1}$

~1-2 orders of magnitude

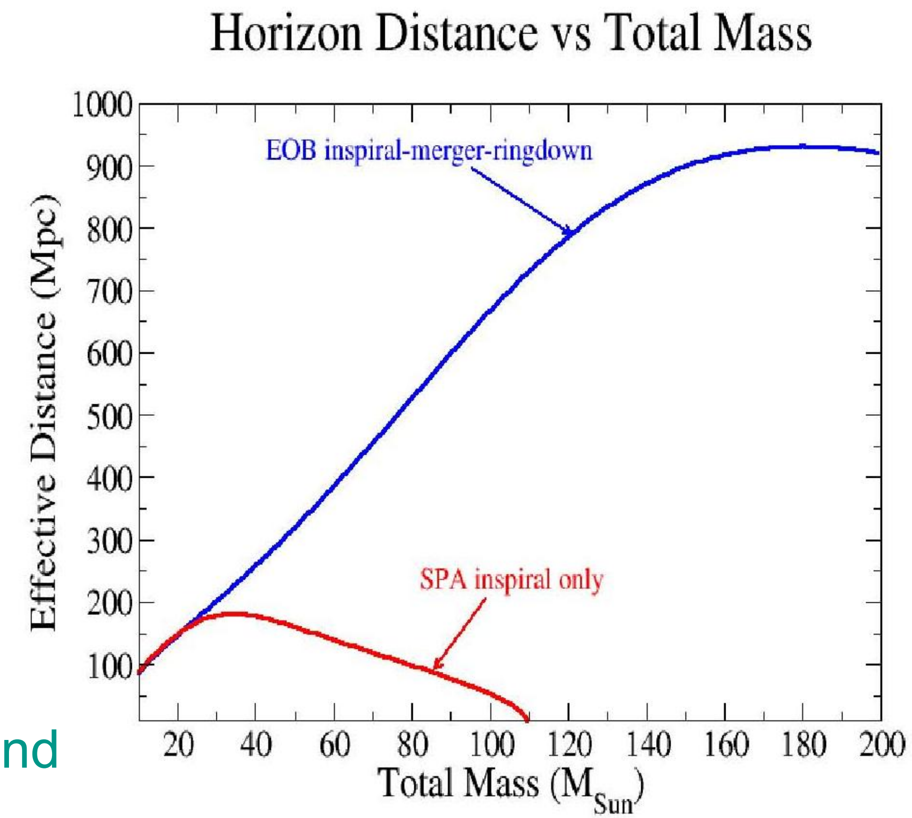
- Astrophysical best estimate rates
  - » BNS rate = $5 \times 10^{-5}\, L_{10}^{-1}\, yr^{-1}$
  - » BBH rate = $4 \times 10^{-7}\, L_{10}^{-1}\, yr^{-1}$
  - » NSBH rate = $2 \times 10^{-6}\, L_{10}^{-1}\, yr^{-1}$

~3 orders of magnitude

High mass search (I)



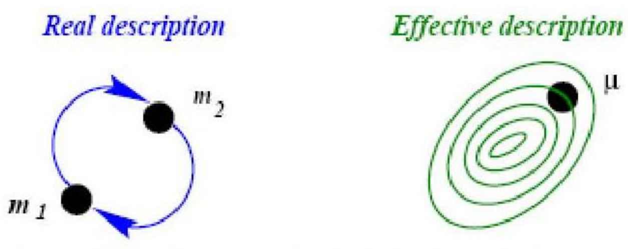
- Inspiral-Merger-Ringdown (IMR) templates to model the entire in-band gravitational wave signal
  - High mass waveforms can be very short (~100 ms). Merger and ringdown are a large part of the in band signal.
  - Effective-One-Body (EOB) model tuned to Numerical Relativity (NR) simulations = EOBNR waveforms

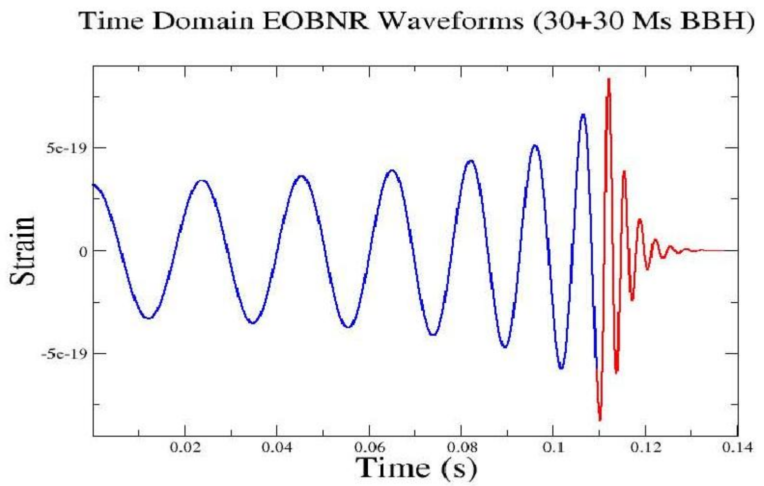


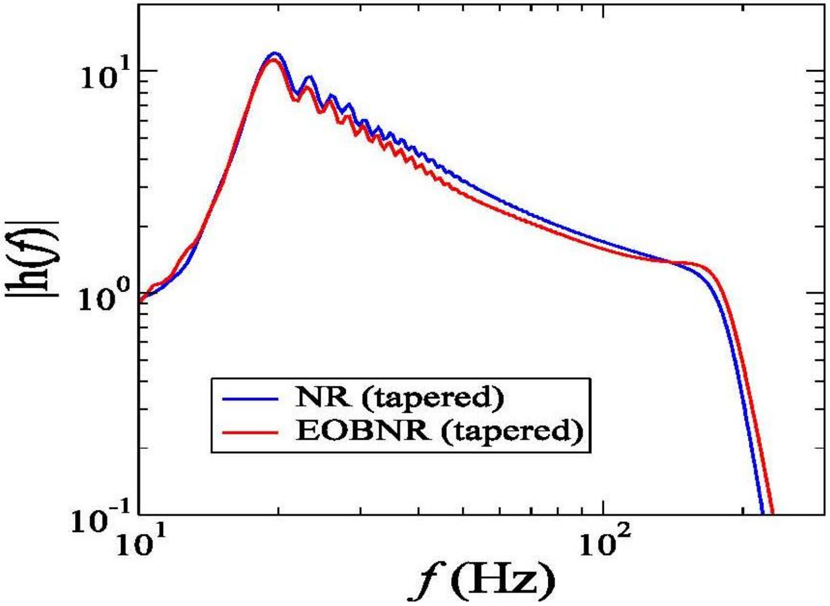
Calculated with analytic noise curve

- Could detect high mass binaries out to several hundred Mpc

High mass search (II)



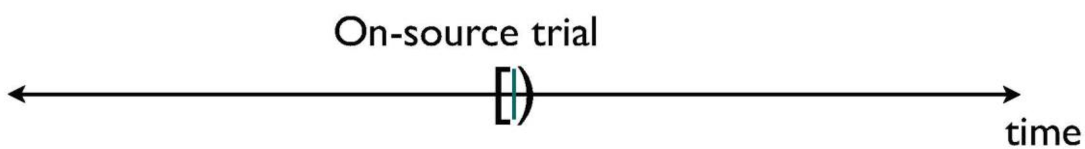




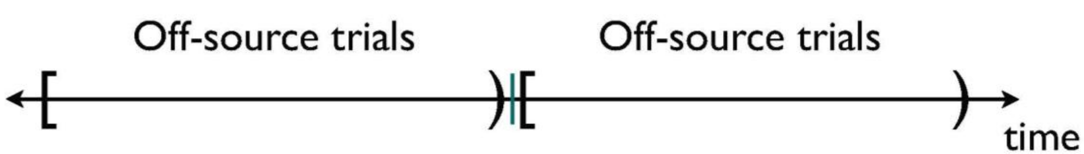
- EOBNR: complete analytic IMR waveforms
- EOB inspiral-plunge waveform computed up to the light ring
- Merger-ringdown waveform: superposition of quasi-normal modes smoothly attached near the light ring
- Model calibrated to NR waveforms with mass ratios 1:1 – 4:1

GRB search (I)

- 22 short GRBs during S5-VSR1 with at least two detectors taking good data
- Known time and sky location
  - » Lower thresholds can be used to dig deeper into the detector noise
- On-source and Off-source
  - » GW triggers associated with GRB within [–5, +1) s of the reported GRB time



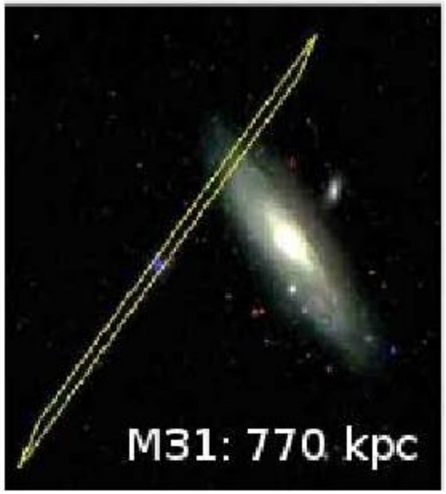
  - » Background estimated from ~40 minutes of nearby data

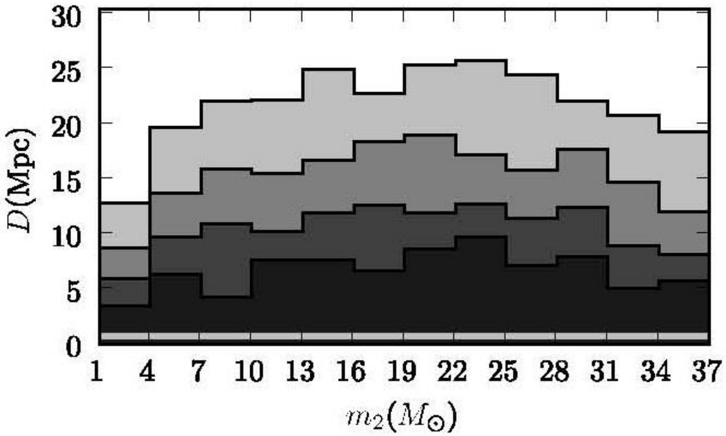


051114	070209
051210	070429B
051211	070512
060121	070707
060313	070714
060427B	070714B
060429	070724
061006	070729
061201	070809
061217	070810B
070201	070923

GRB search (II)

- Example from already published GRB 070201
  - » Error box intersecting M31
  - » Merger in Andromeda?
  - » No plausible GW signal found
  - » 1 $M_\odot$ < m_1 < 3 $M_\odot$ and 1 $M_\odot$ < m_2 < 40 $M_\odot$ excluded at 99% confidence





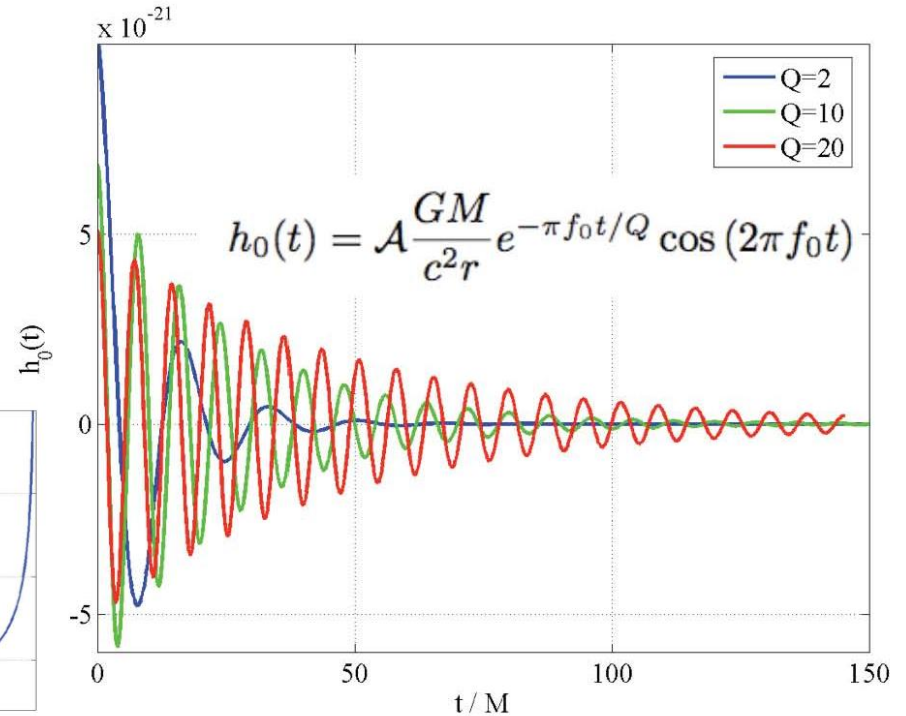
[Ref.17]

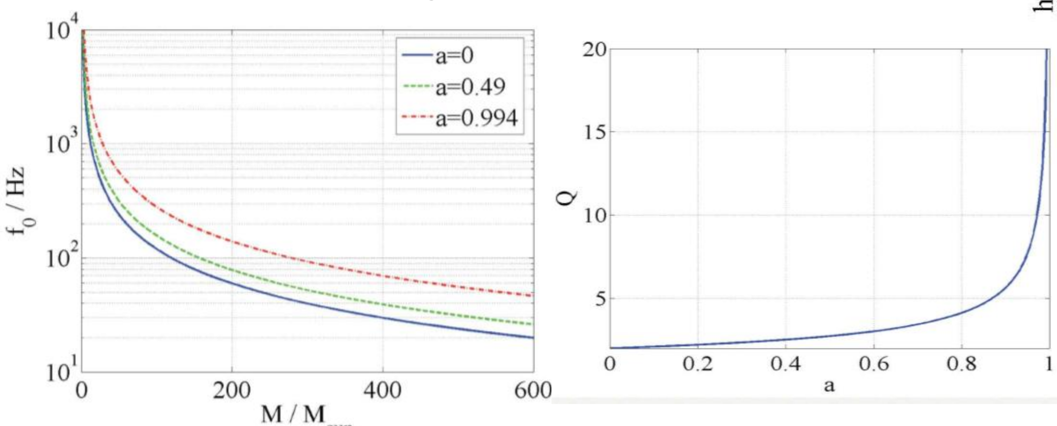
- Similar exclusion plots derived for all analyzed S5-VSR1 short GRBs
- Population statement combining results from all GRBs also derived

Ringdown search

- Late stage of coalescence
  - Perturbed black hole returning to equilibrium through emission of ringdown GW signal
  - Superposition of quasi-normal modes
  - Waveform determined by mass M and spin $\hat{a} = \frac{Jc}{GM^2}$ of black hole

Exponentially damped sinusoid





$$f_0 = \frac{1}{2\pi}\frac{c^3}{GM}g(a)$$

$$Q = 2\,(1-\hat{a})^{-9/20}$$

where $g(a) = 1 - 0.63\,(1-a)^{3/10}$

[Ref.20]

- S4 upper limit
  - Rate of ringdowns from black holes
  - $R_{90\%} = 1.6 \times 10^{-3}\ L_{10}^{-1}\ yr^{-1}$ in mass range 85-390 $M_\odot$

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