Correction of Doppler Effect by Discrete Signal Resampling

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Introduction

The technique

Simulation

Technique validation

Possible application to VSR1

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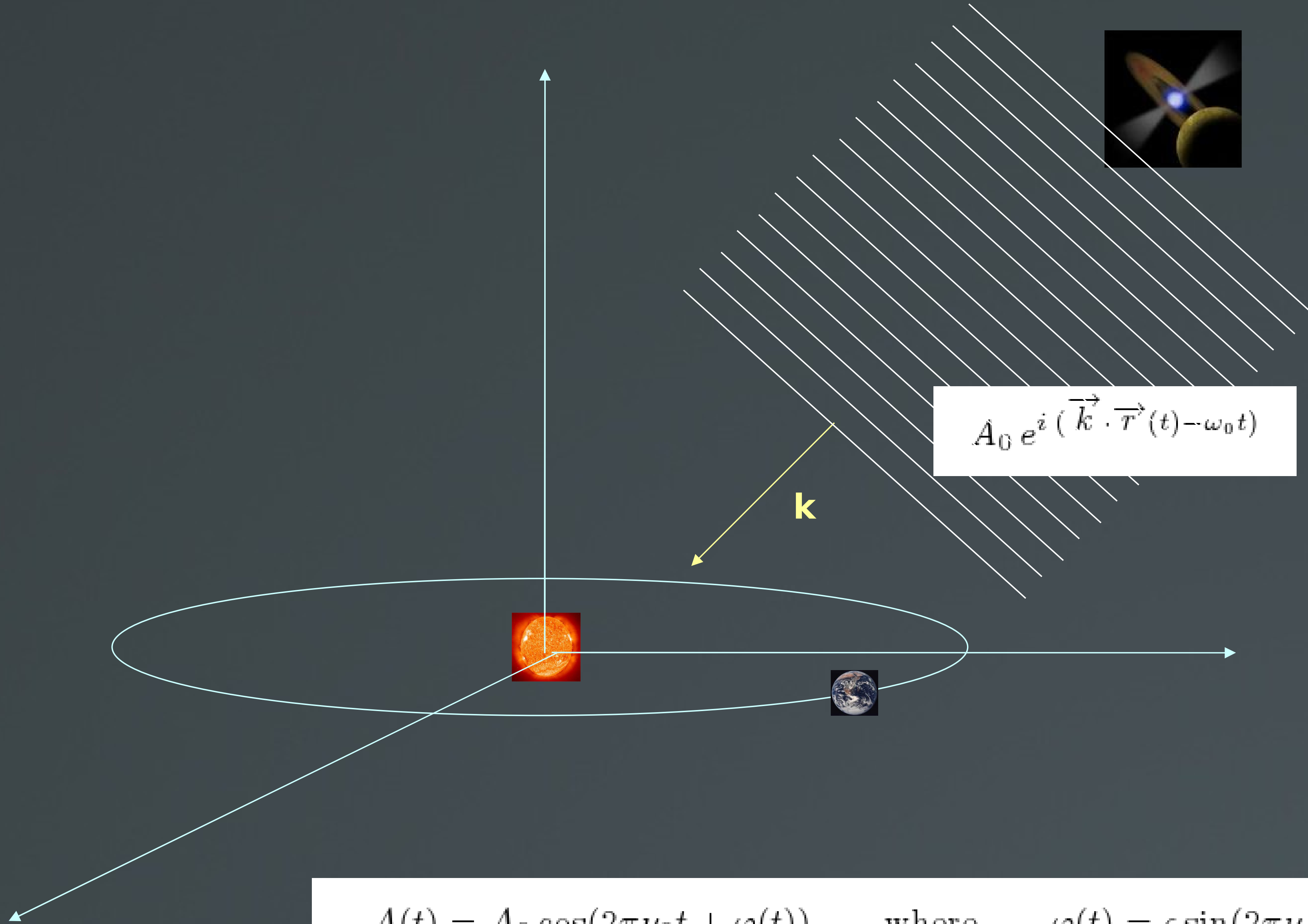
Possible application to VSR1



$$A_0\, e^{i\,(\vec{k}\cdot\vec{r}(t)-\omega_0 t)}$$

$$A(t) = A_0 \cos(2\pi\nu_0 t + \varphi(t)) \qquad \text{where} \qquad \varphi(t) = \epsilon \sin(2\pi\nu_y t)$$

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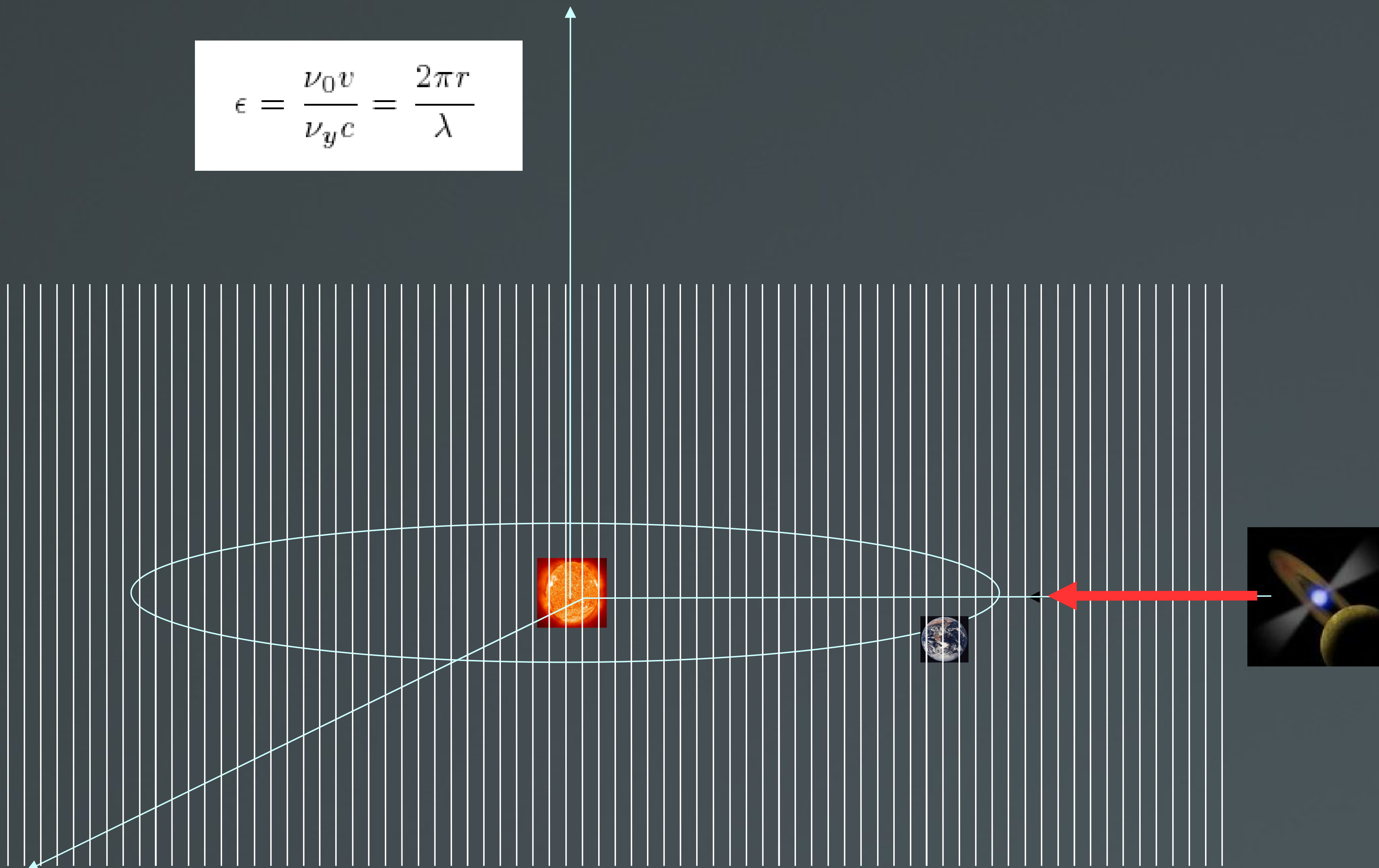
Source on the orbital plane

$$\epsilon = \frac{\nu_0 v}{\nu_y c} = \frac{2\pi r}{\lambda}$$

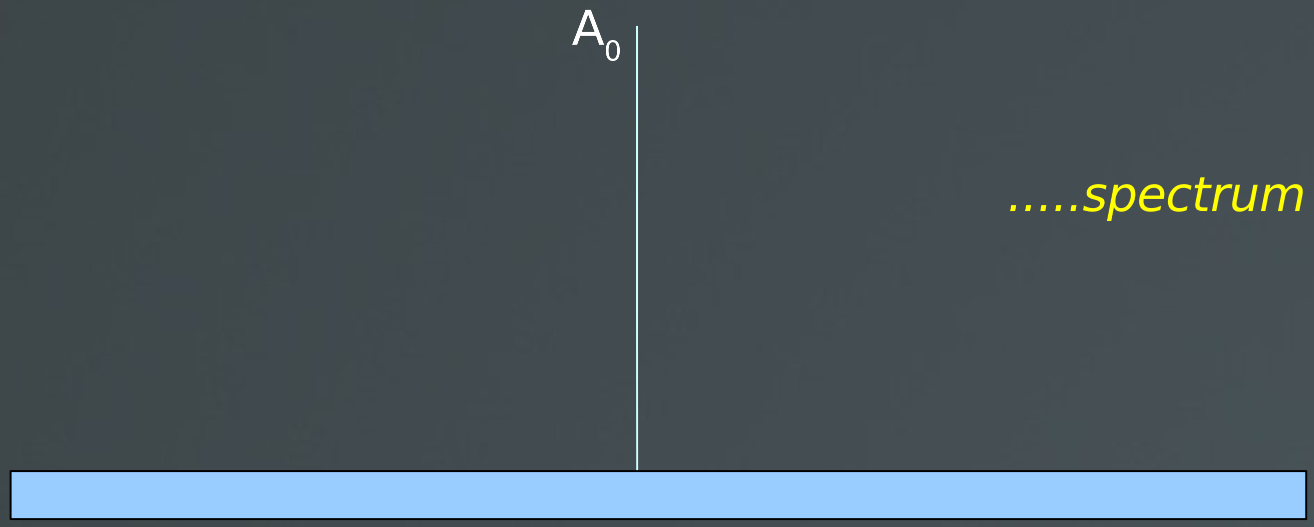


As for any phase modulation signal energy is conserved, but spread on a wide band ($2\beta\nu_0 = 2\varepsilon\nu_y$)

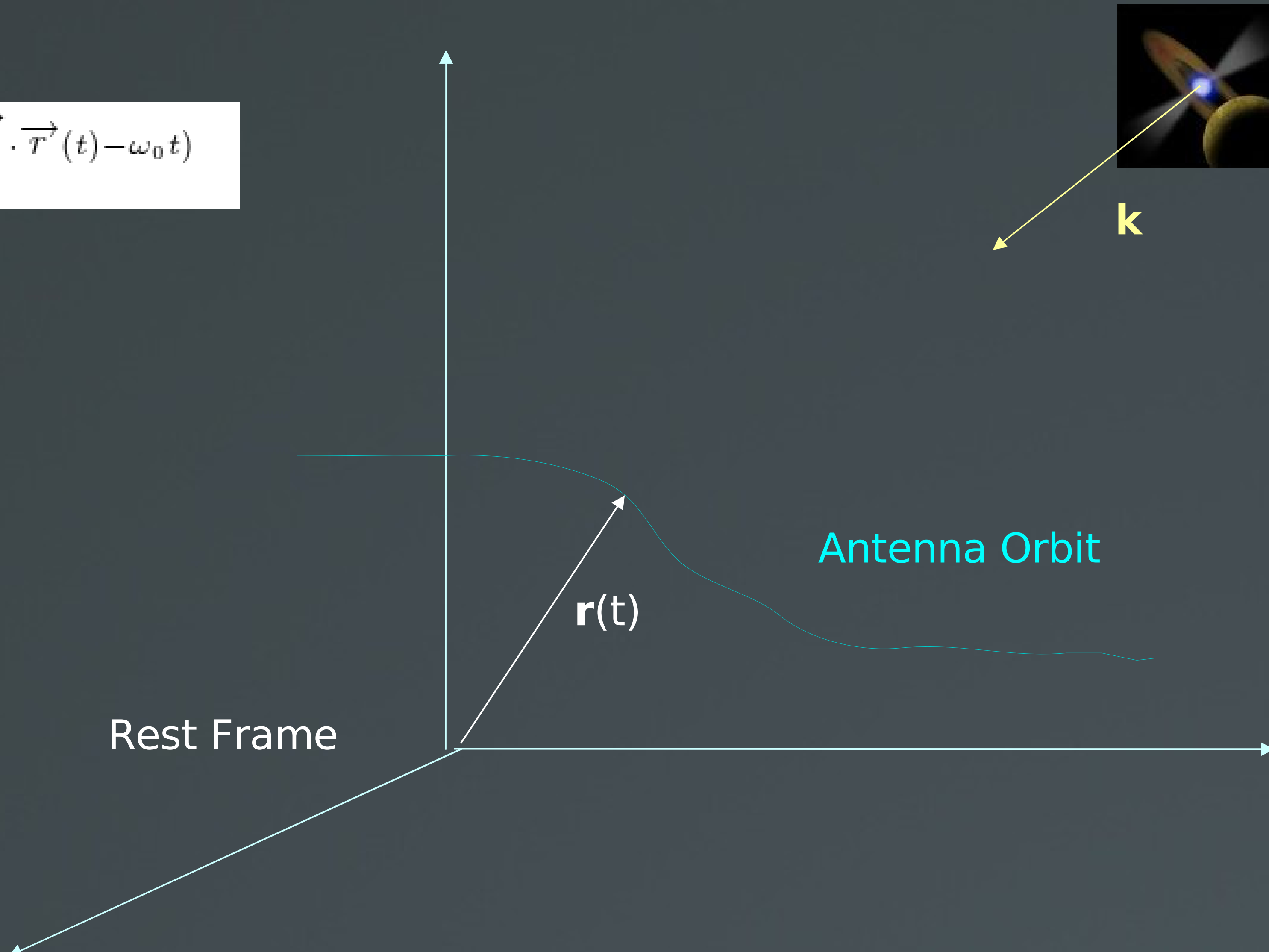
$$\frac{A_1^2}{A_0^2} = \frac{\delta\nu}{\Delta\nu} = \frac{\delta\nu}{2\beta\nu_0} \simeq \frac{3.17\ 10^{-8}}{2\ 10^{-4}\ 10^{2}} \simeq 1.6\ 10^{-6}$$

58 dB reduction of the signal due to the spread…….

…..spectrum drown in the noise floor



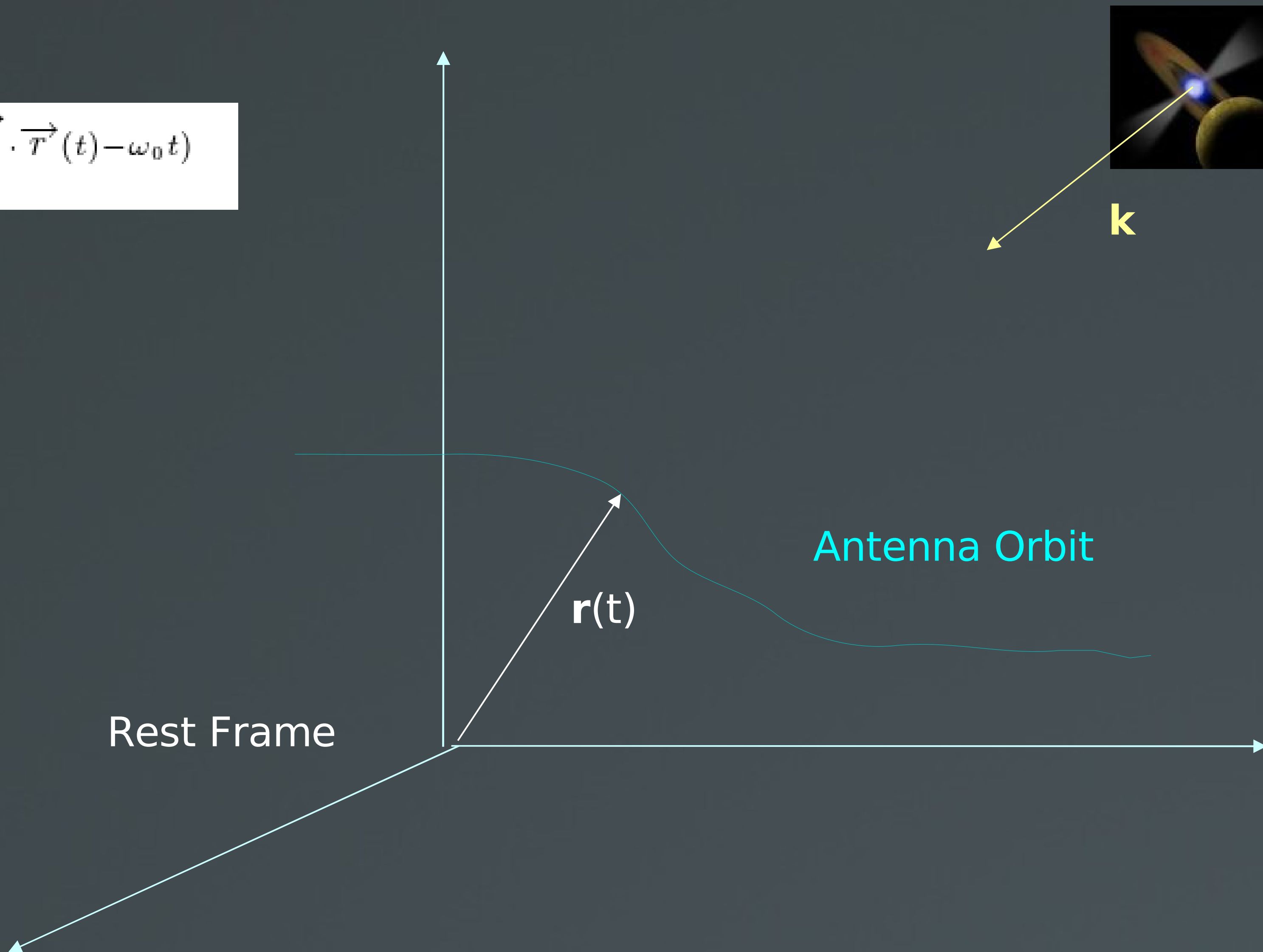
$$A_0\, e^{i\,(\overrightarrow{k}\cdot\overrightarrow{r}(t)-\omega_0 t)}$$



x $e^{-i(\overrightarrow{k}\cdot\overrightarrow{r}(t))}$

Recover the rest frame signal

$$A_0\, e^{i\,(\overrightarrow{k}\cdot\overrightarrow{r}(t)-\omega_0 t)}$$



x $e^{-i(\overrightarrow{k}\cdot\overrightarrow{r}(t))}$

Recover the rest frame signal

Residual modulation dominated by the uncertain on direction

$$A(t) = A_0 \cos(\omega_0 t + \epsilon \sin(\omega_y t)) = A_0 \sum_{-\infty}^{+\infty} J_n(\epsilon) \cos(\omega_0 + n\omega_y)t$$

$\varepsilon = 1$ *rad* →

$$J_0 = 0.765198$$
$$J_1 = 0.440051$$
$$J_2 = 0.114903$$
$$J_3 = 0.019563$$
$$J_4 = 0.002477$$
$$\ldots\ldots$$

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Phase Modulation depth of 1 rad is enough to maintain a large part of the spectral energy around the main frequency

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Phase Modulation depth of 1 rad is enough to maintain a large part of the spectral energy around the main frequency

Very far from this goal.....

$$\epsilon = \frac{\nu_0 v}{\nu_y c} = \frac{2\pi r}{\lambda}$$

Earth position has to be known better than $\lambda/2\pi$

500 km for a 100 Hz signal --> no problem for JPL (hundreds meters)

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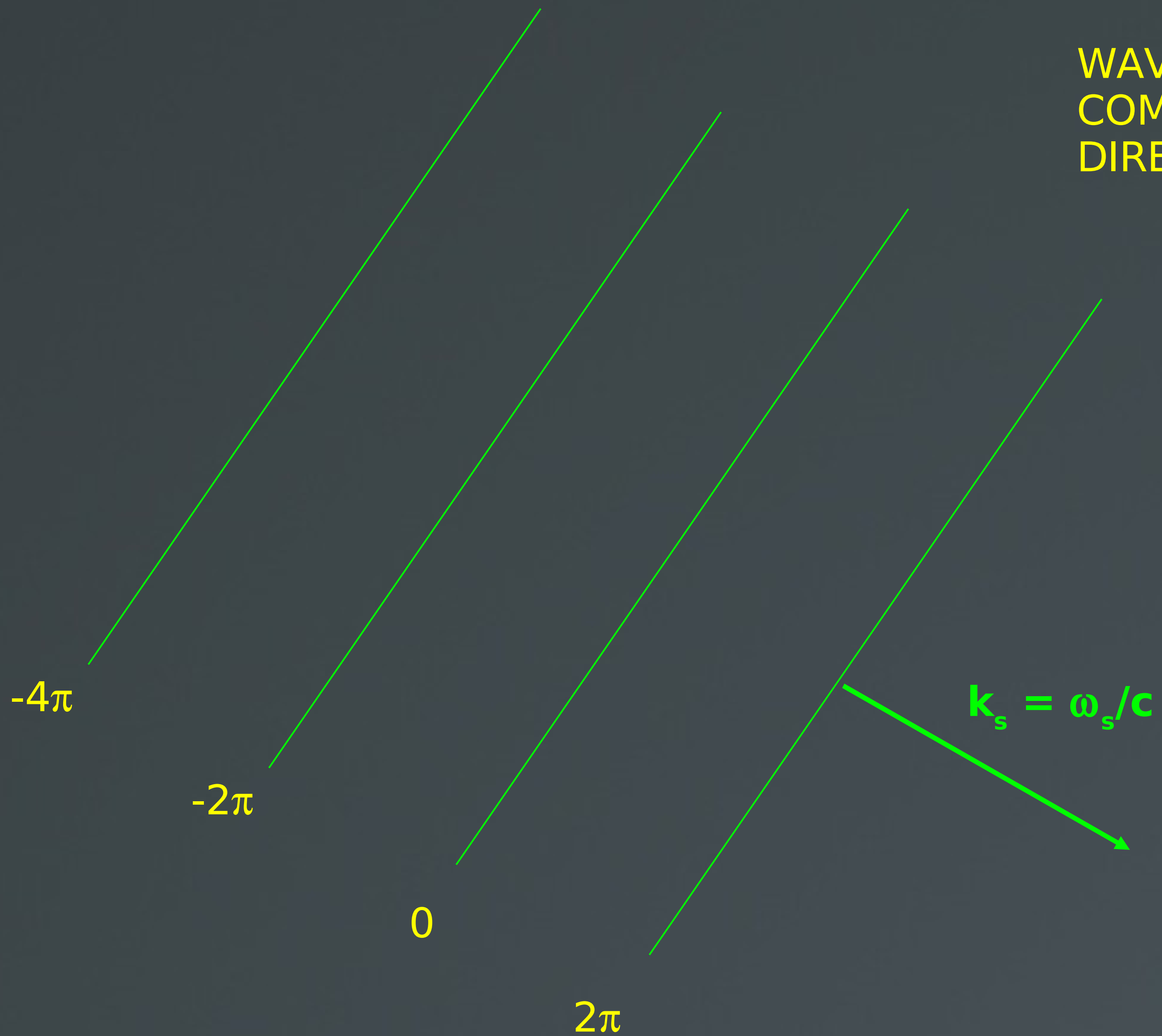
In 1 year integration time I cannot loose 1 rad

$$\frac{1}{2\pi 100 \cdot 1\ year}$$

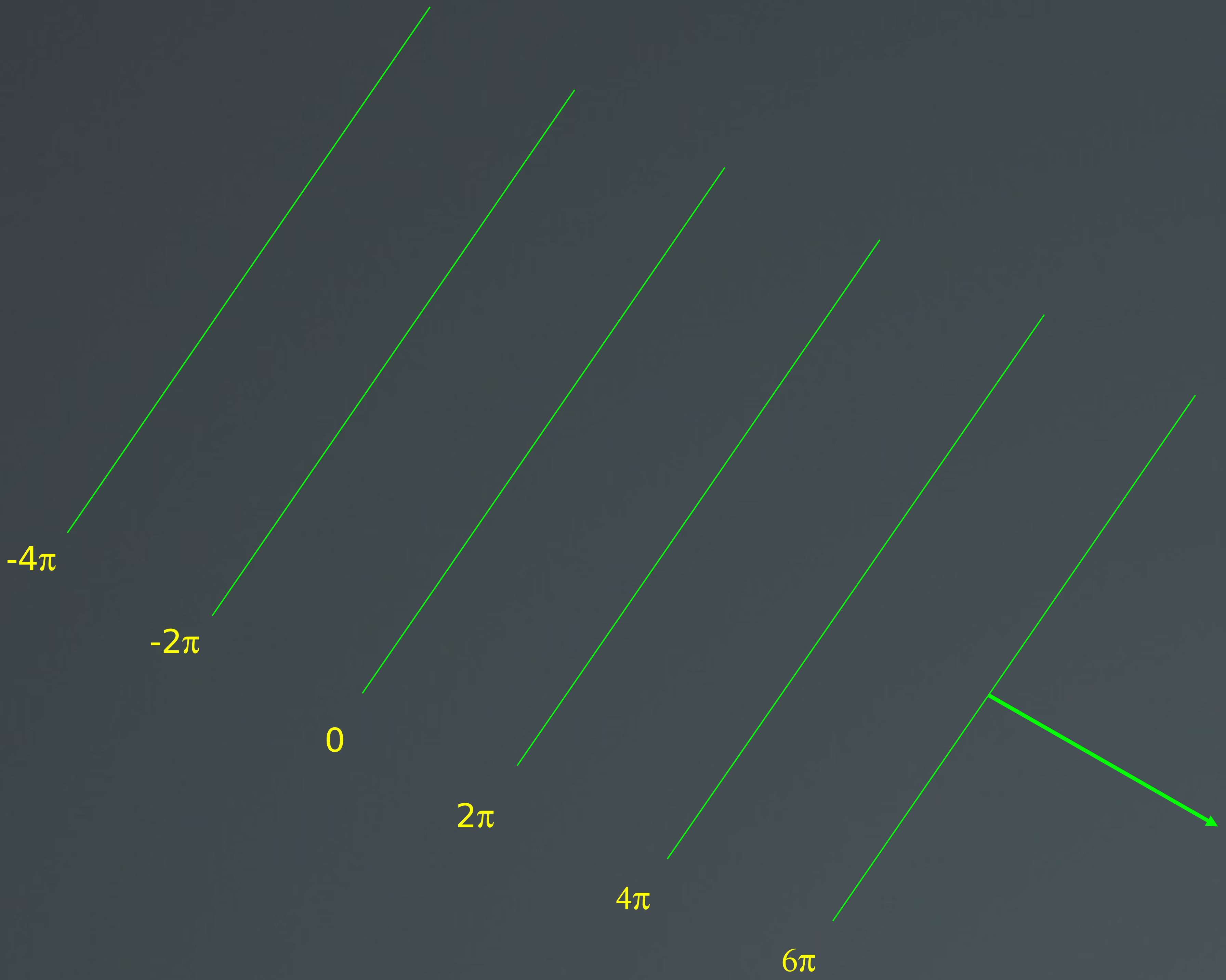
Clock stability of 2 parts on 10^{10}
No problem for VIRGO locked to GPS

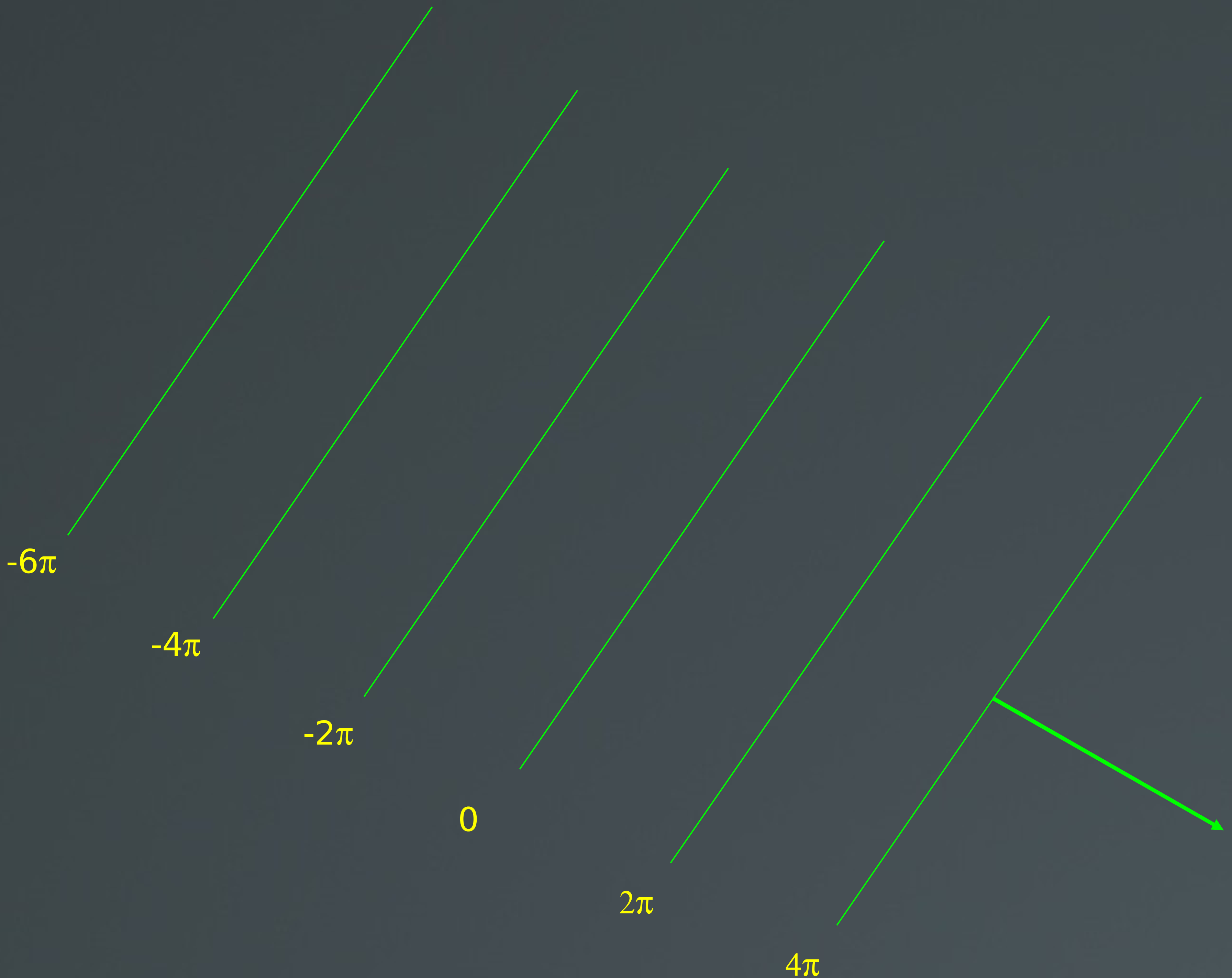
Clock stability and Earth Position measurement accuracy are enough

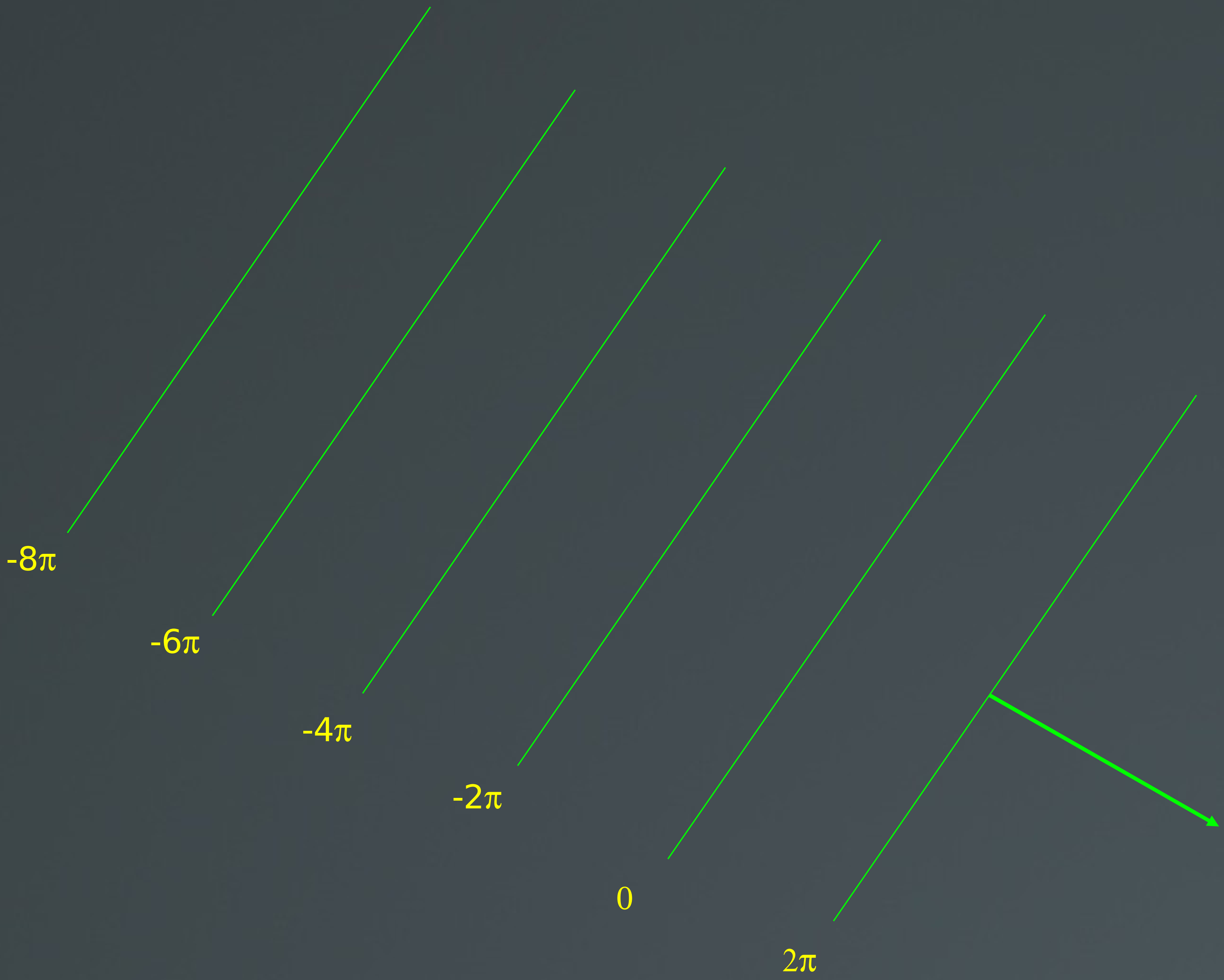
WAVE AT SAMPLING FREQUENCY COMING FROM A GIVEN DIRECTION....

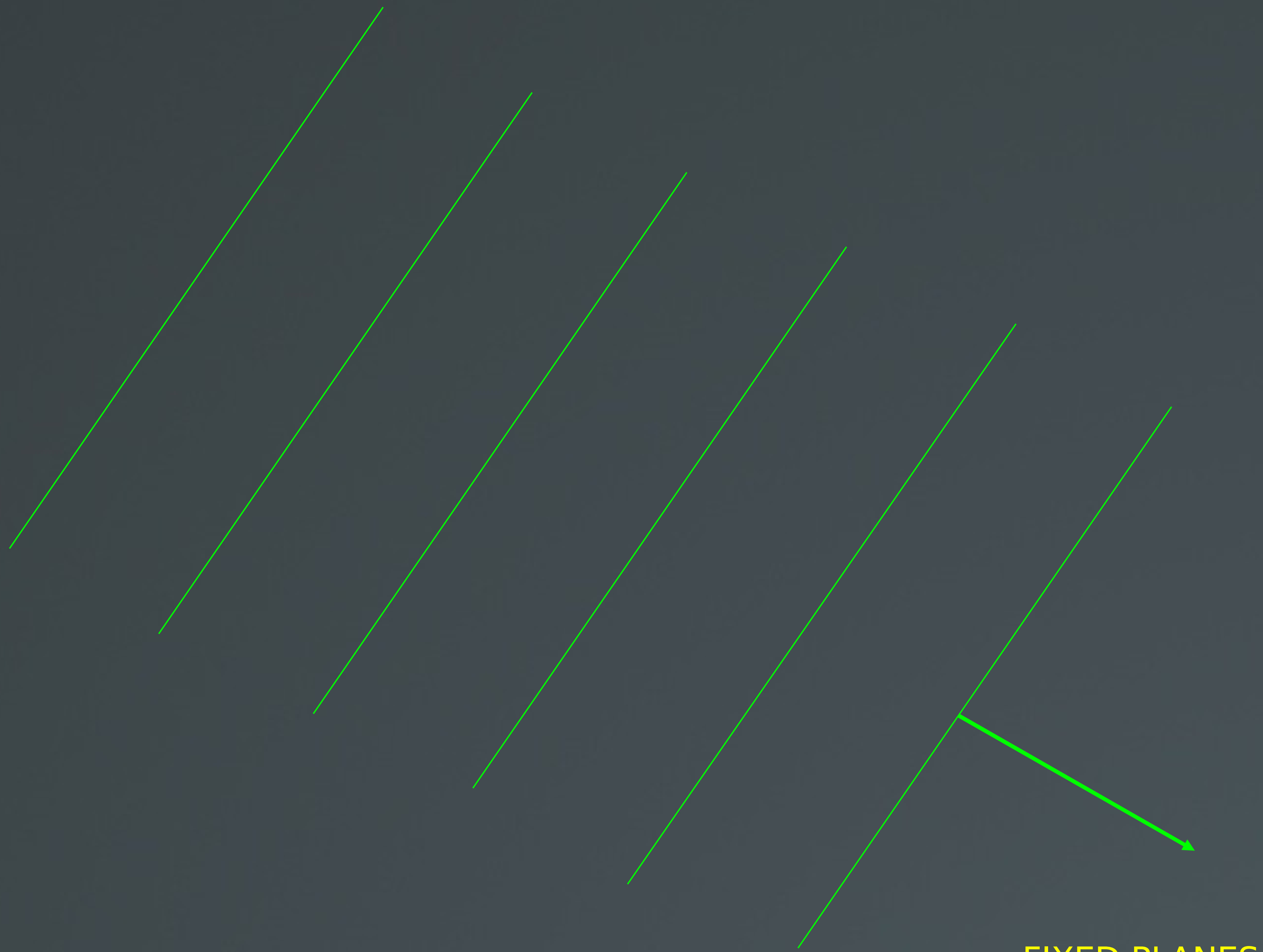


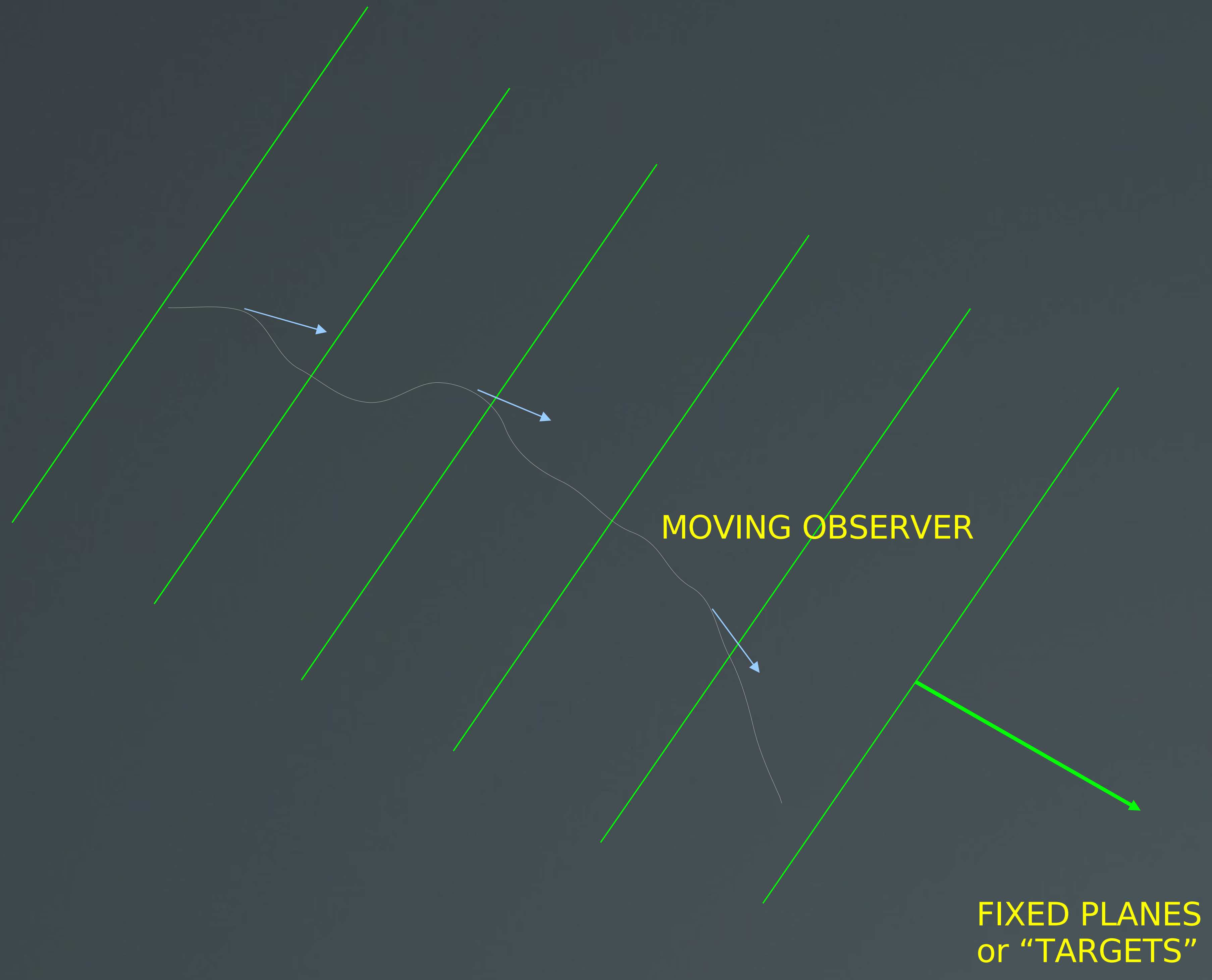
Family of equi-phase plane travelling at *c* at a phase distance 2π or $\omega_s \Delta t$

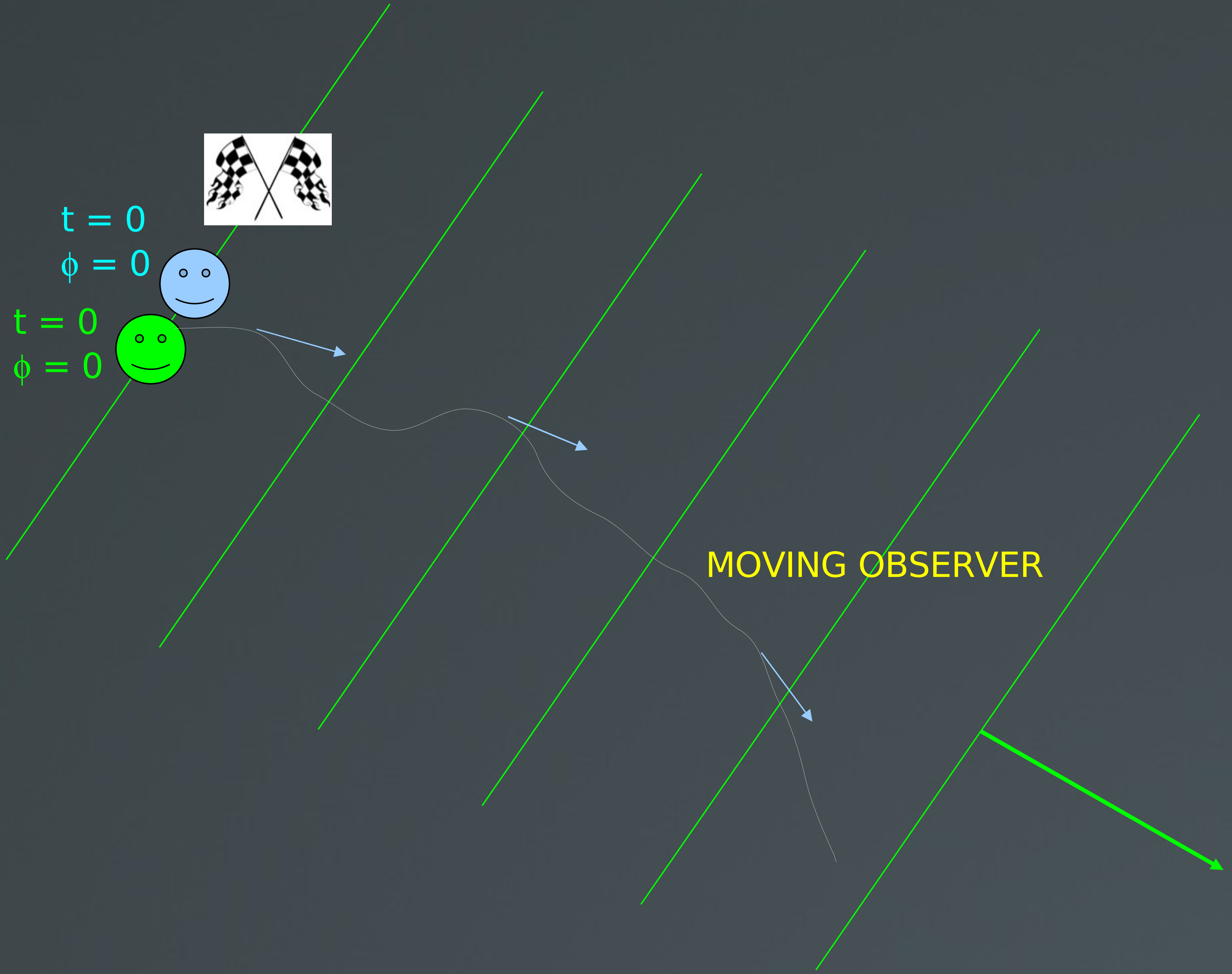

-4π
-2π
0
2π
4π
6π

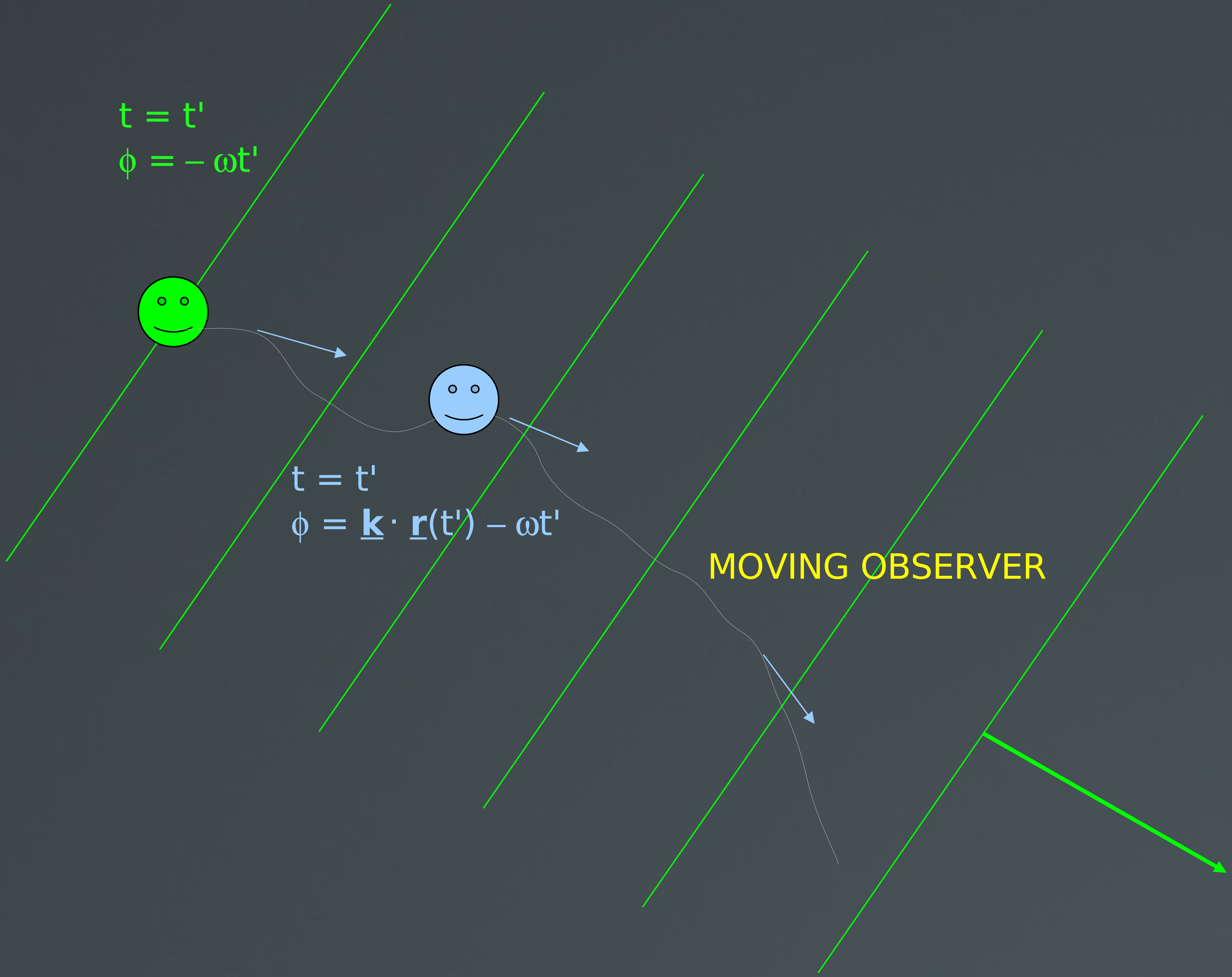

-6π
-4π
-2π
0
2π
4π

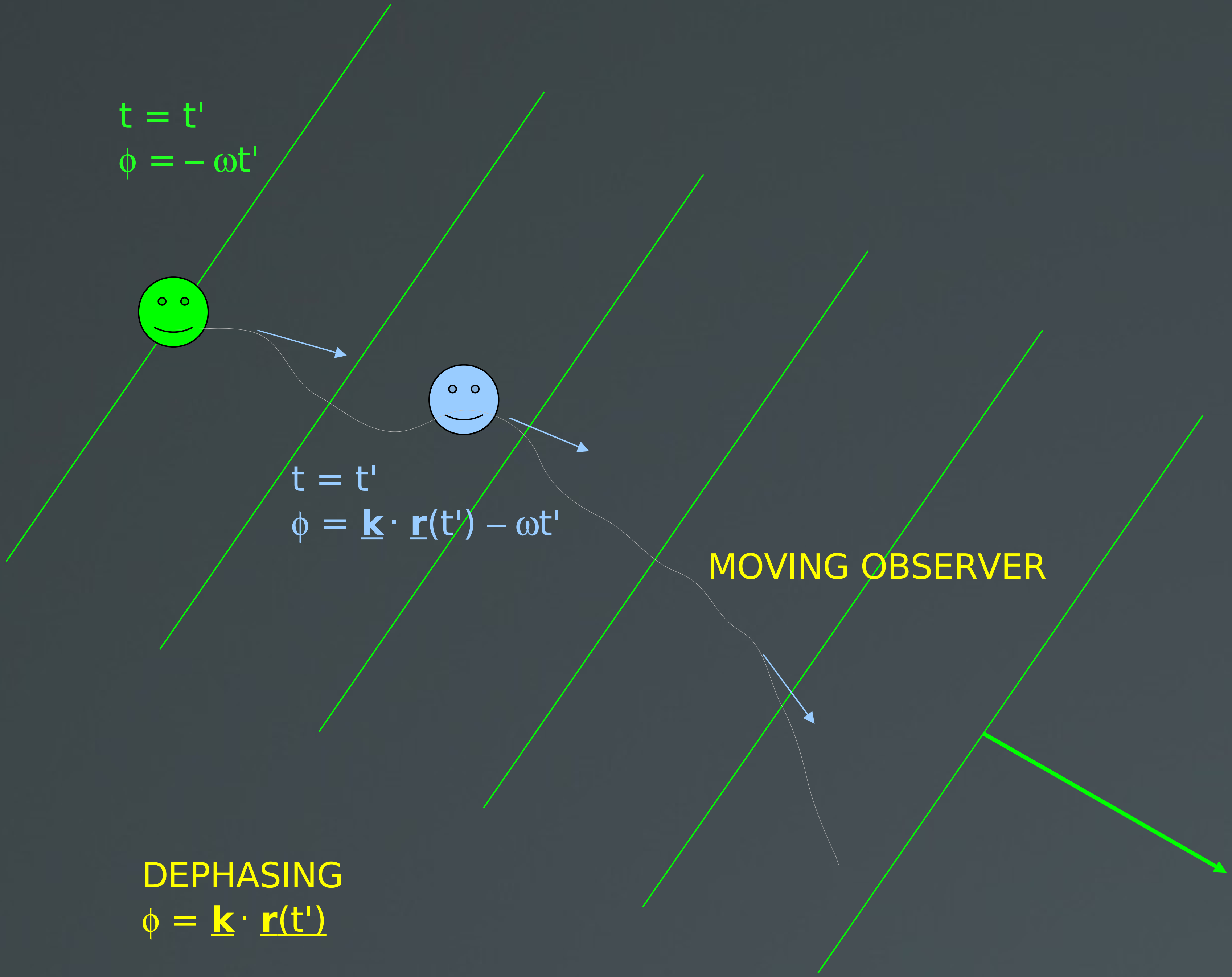

-8π
-6π
-4π
-2π
0
2π

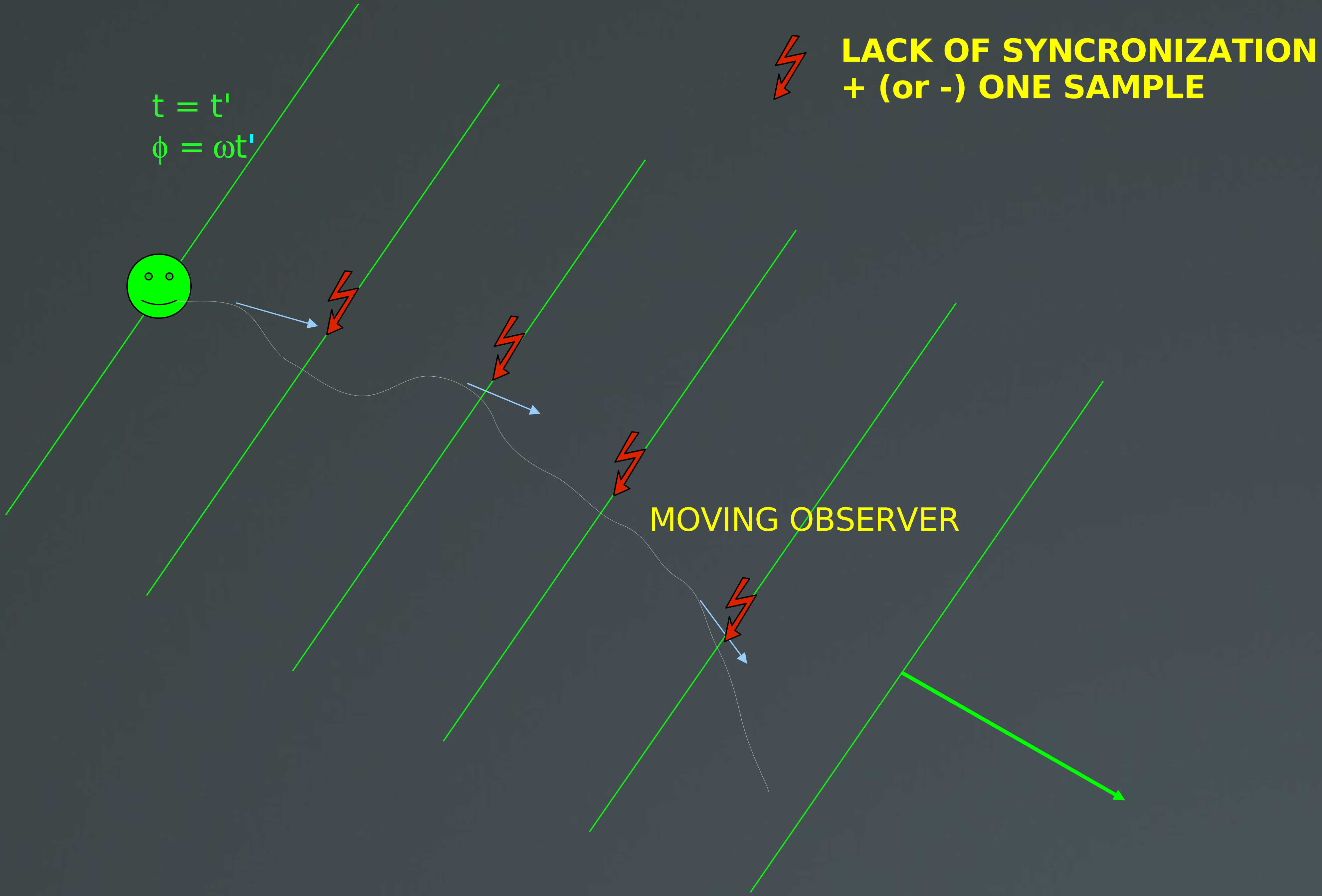

FIXED PLANES
or “TARGETS”

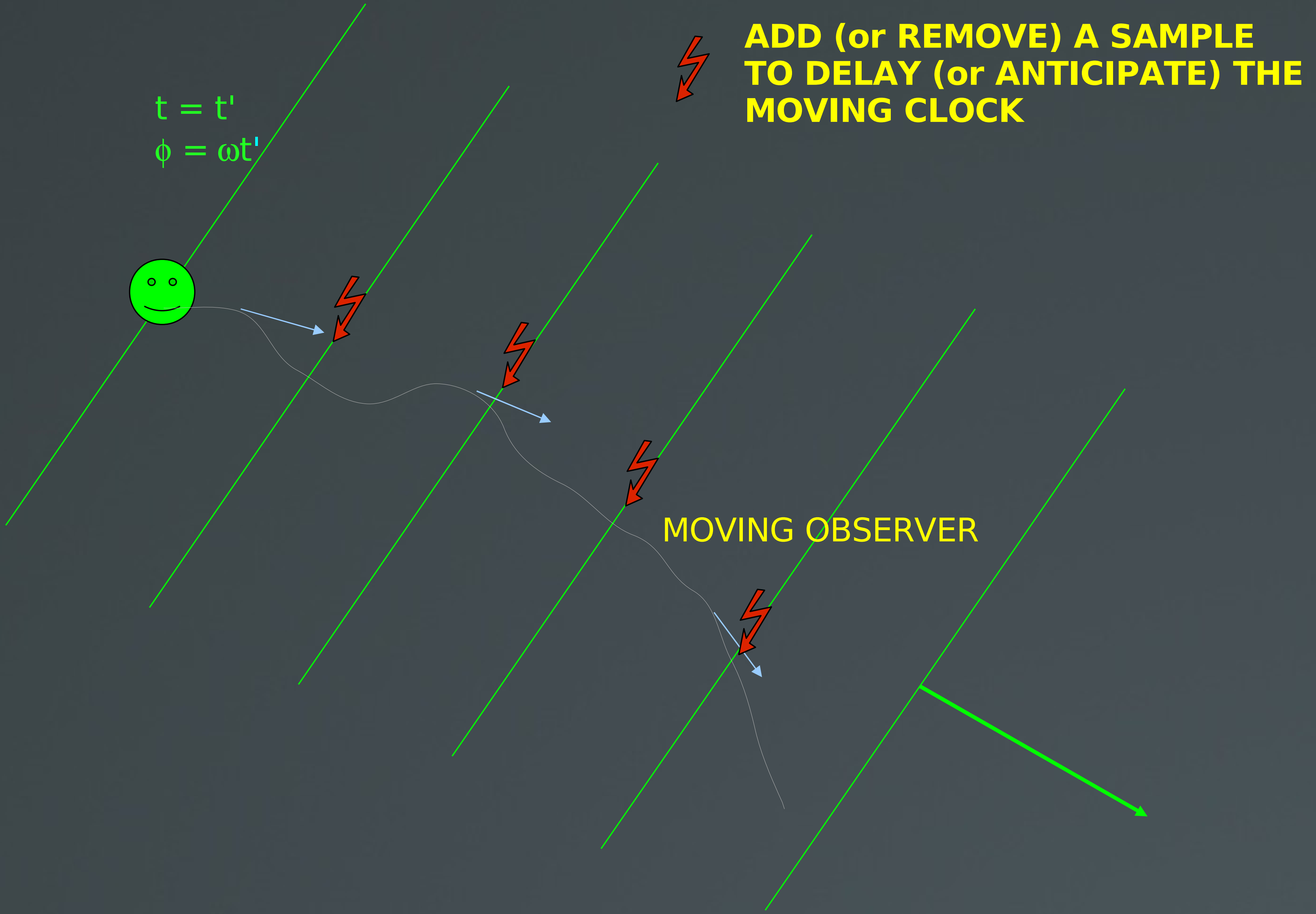

MOVING OBSERVER
FIXED PLANES
or “TARGETS”


t = 0
ϕ = 0
t = 0
ϕ = 0
MOVING OBSERVER

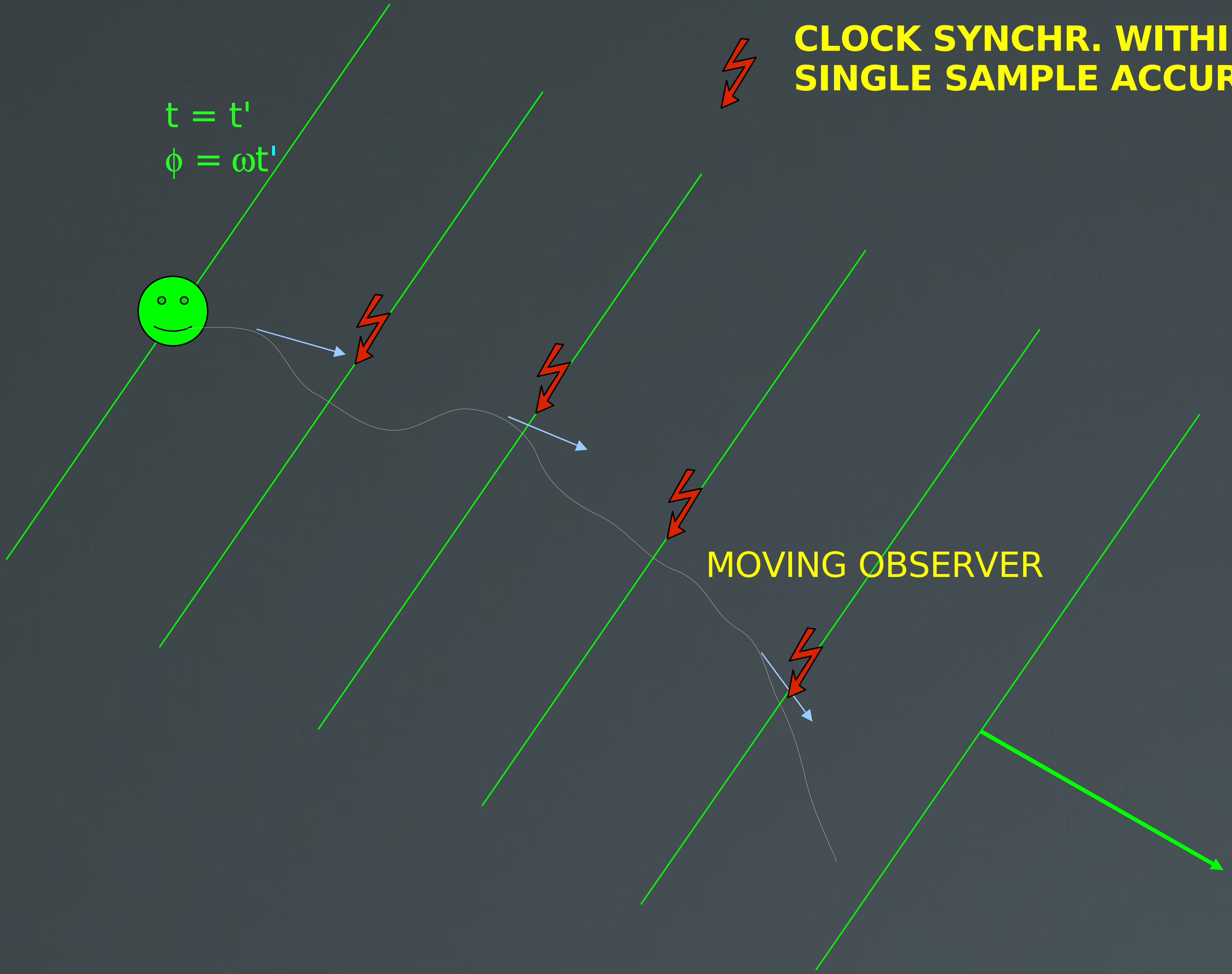

t = t'
ϕ = − ωt'
t = t'
ϕ = k · r(t') − ωt'
MOVING OBSERVER


t = t'
ϕ = – ωt'
t = t'
ϕ = k · r(t') – ωt'
MOVING OBSERVER
DEPHASING
ϕ = k · r(t')


LACK OF SYNCRONIZATION
+ (or -) ONE SAMPLE
t = t'
ϕ = ωt'
MOVING OBSERVER



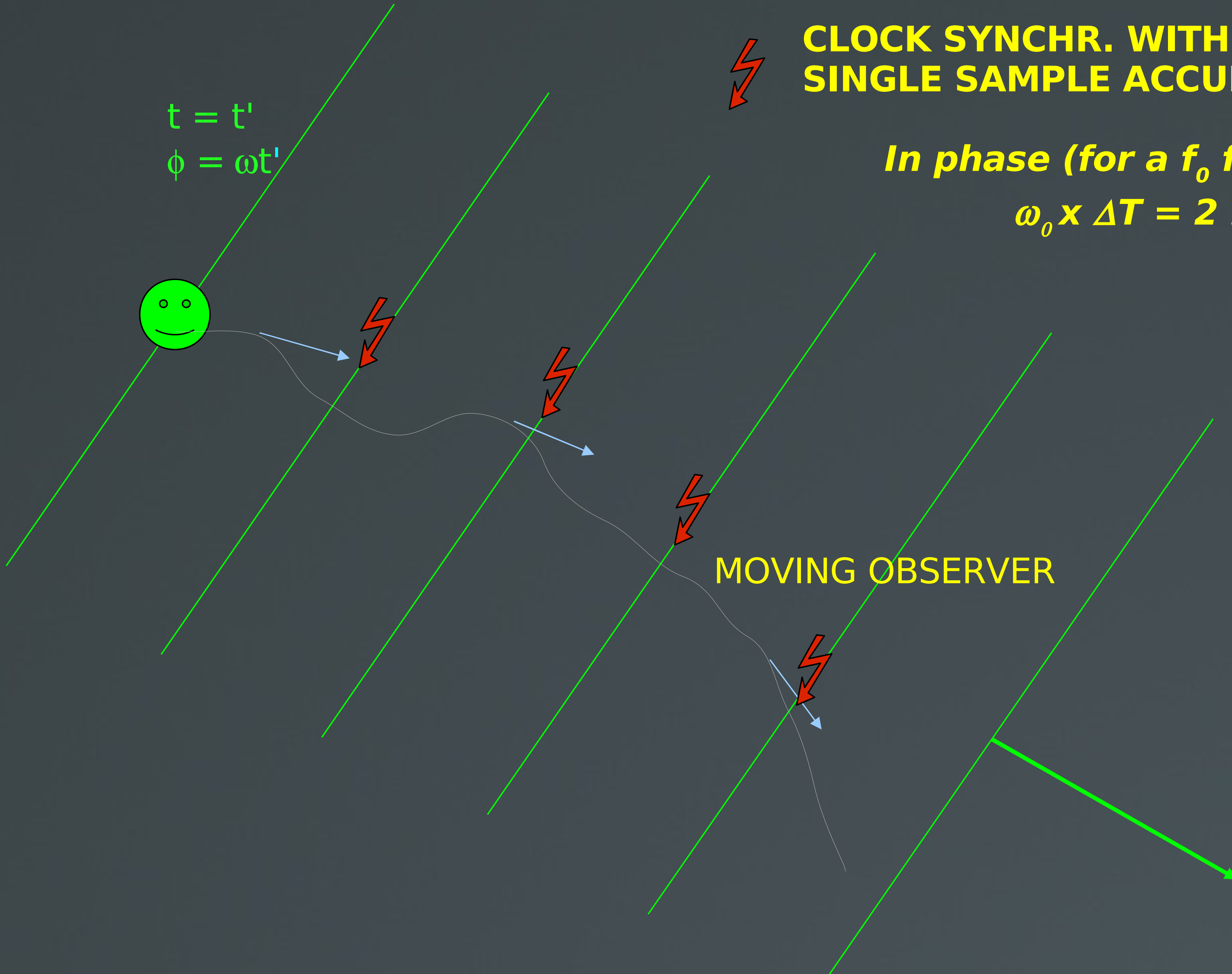
ADD (or REMOVE) A SAMPLE
TO DELAY (or ANTICIPATE) THE
MOVING CLOCK
t = t'
ϕ = ωt'
MOVING OBSERVER

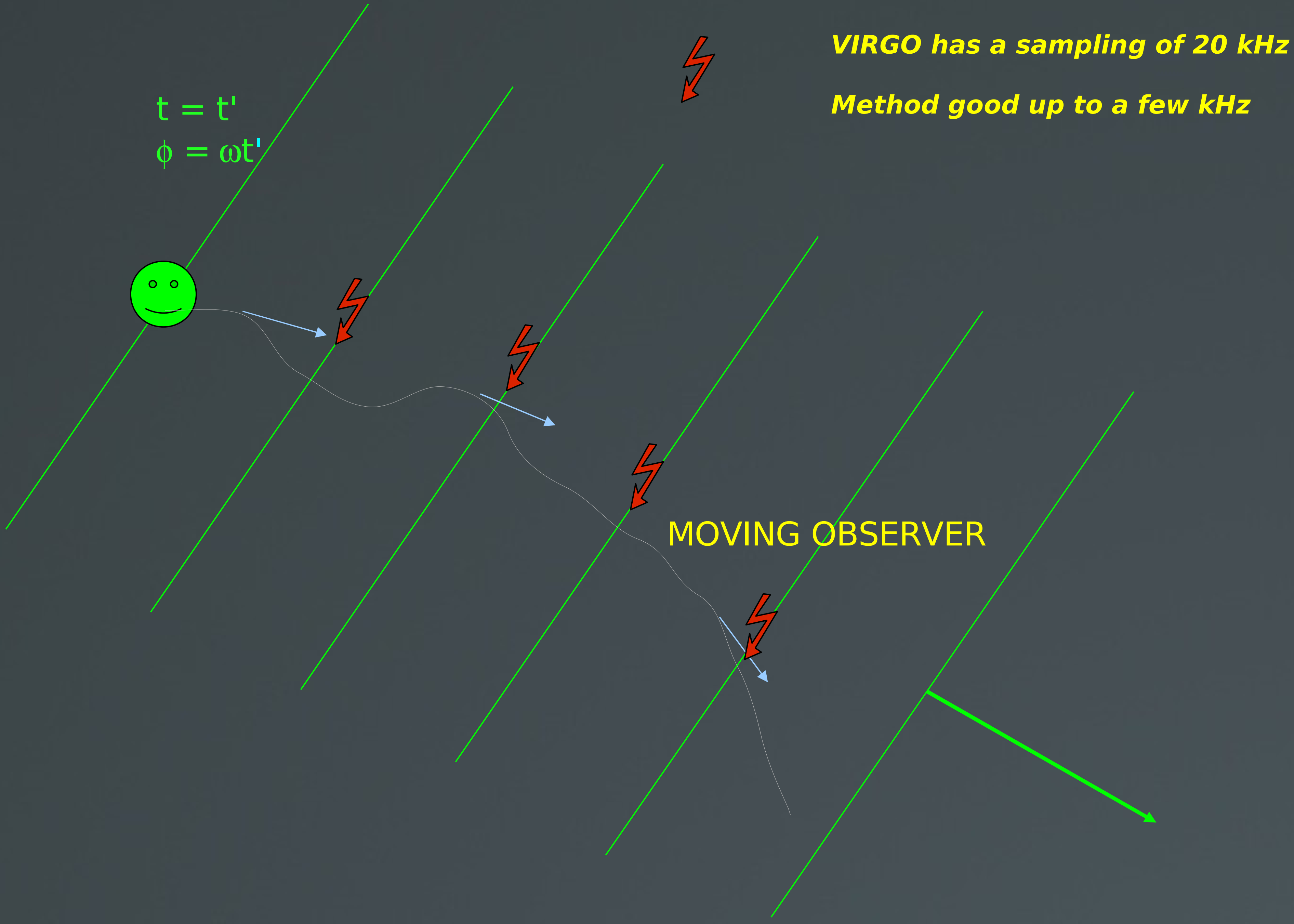

CLOCK SYNCHR. WITHIN A
SINGLE SAMPLE ACCURACY
t = t'
ϕ = ωt'
MOVING OBSERVER

CLOCK SYNCHR. WITHIN A SINGLE SAMPLE ACCURACY

In phase (for a f_0 freq signal)

$$\omega_0 \times \Delta T = 2 \pi f_0 / f_s$$

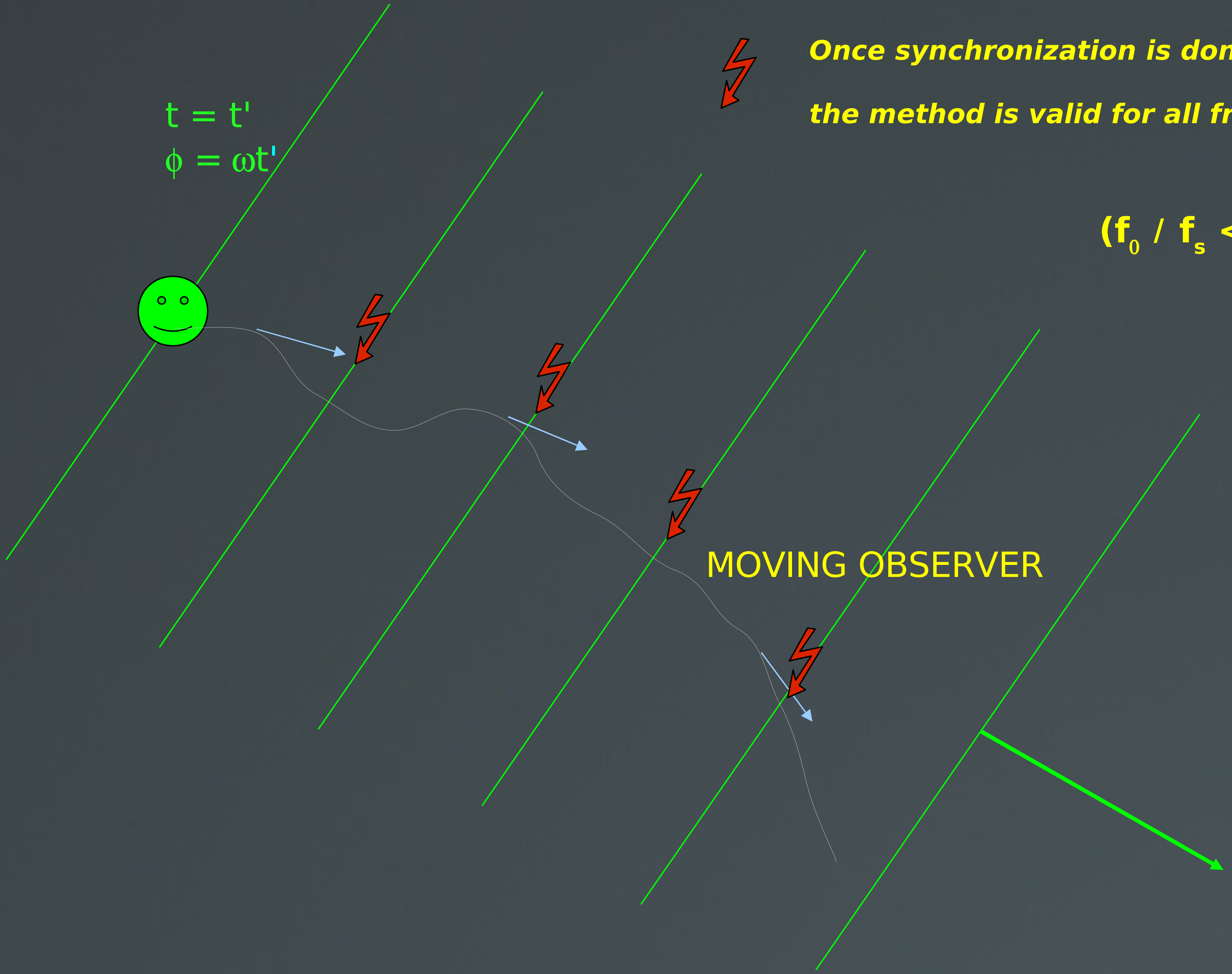



VIRGO has a sampling of 20 kHz
Method good up to a few kHz
t = t'
ϕ = ωt'
MOVING OBSERVER

Once synchronization is done

the method is valid for all frequencies !

(f_0 / f_s < 2 π)



A) GENERATOR OF SIGNALS

$$A = \sin(-2\pi\nu_0 i\ \Delta t)$$

Rest Observer Signal

$$A = \sin(2\pi\nu_0\ (\frac{n_x x + n_y y + n_z z}{c} - i\Delta t))$$

Moving Observer Signal

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Moving Observer Signal

B) MASK ROUTINE

Computes all times (expressed in rest frame sample index) a crossing of a target plane occur and associate to them a 1 or -1 flag (add, delete). A two column file is prepared

Action_Sample *Action_Flag*

Mask.dat file

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Linear approximation $(c\Delta t) / (|\vec{v} \cdot \vec{n}|)$

$$t_{crossing} = 1/(|\vec{\beta} \cdot \vec{n}|\ \nu_s)$$

A) GENERATOR OF SIGNALS

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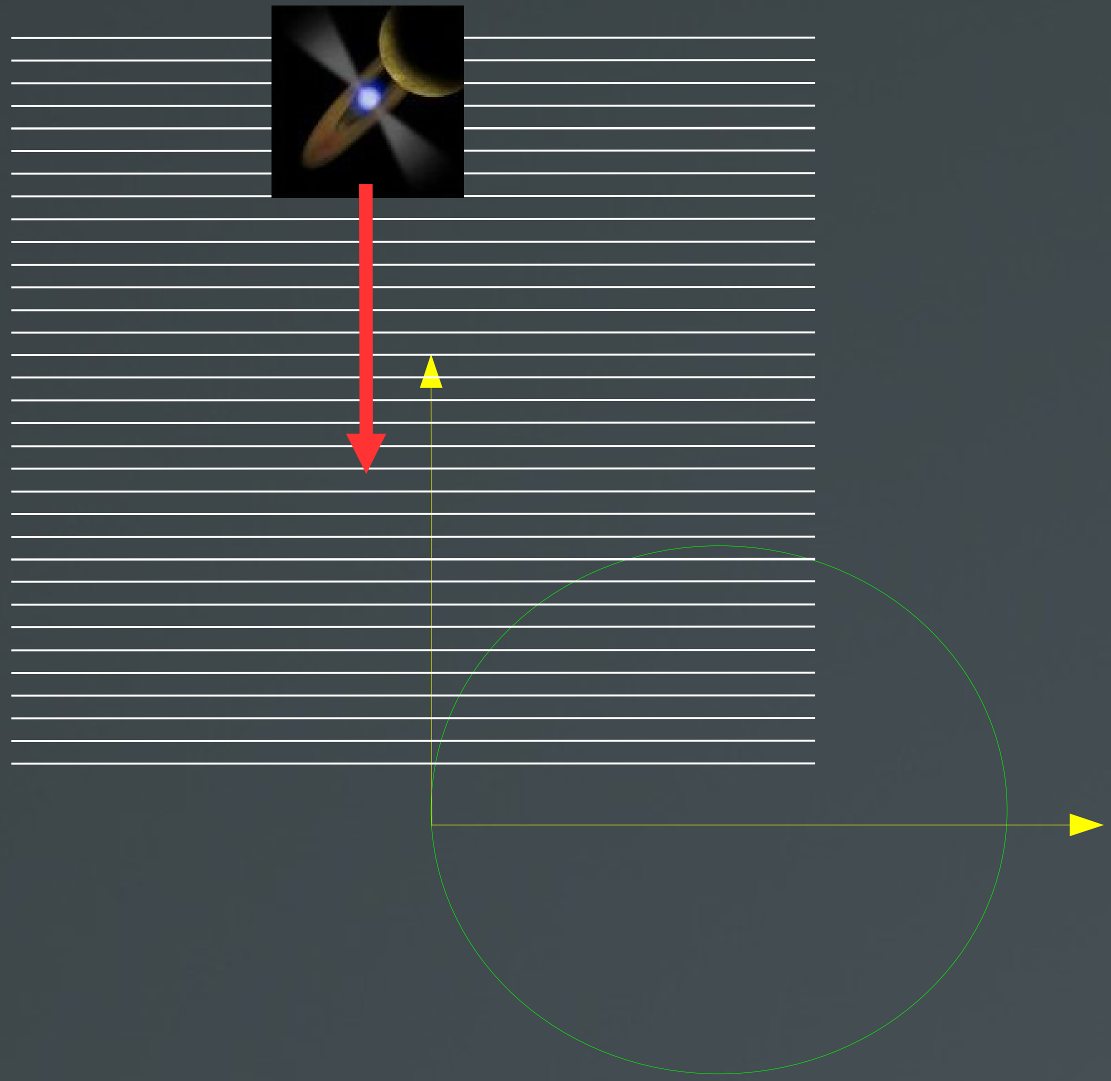
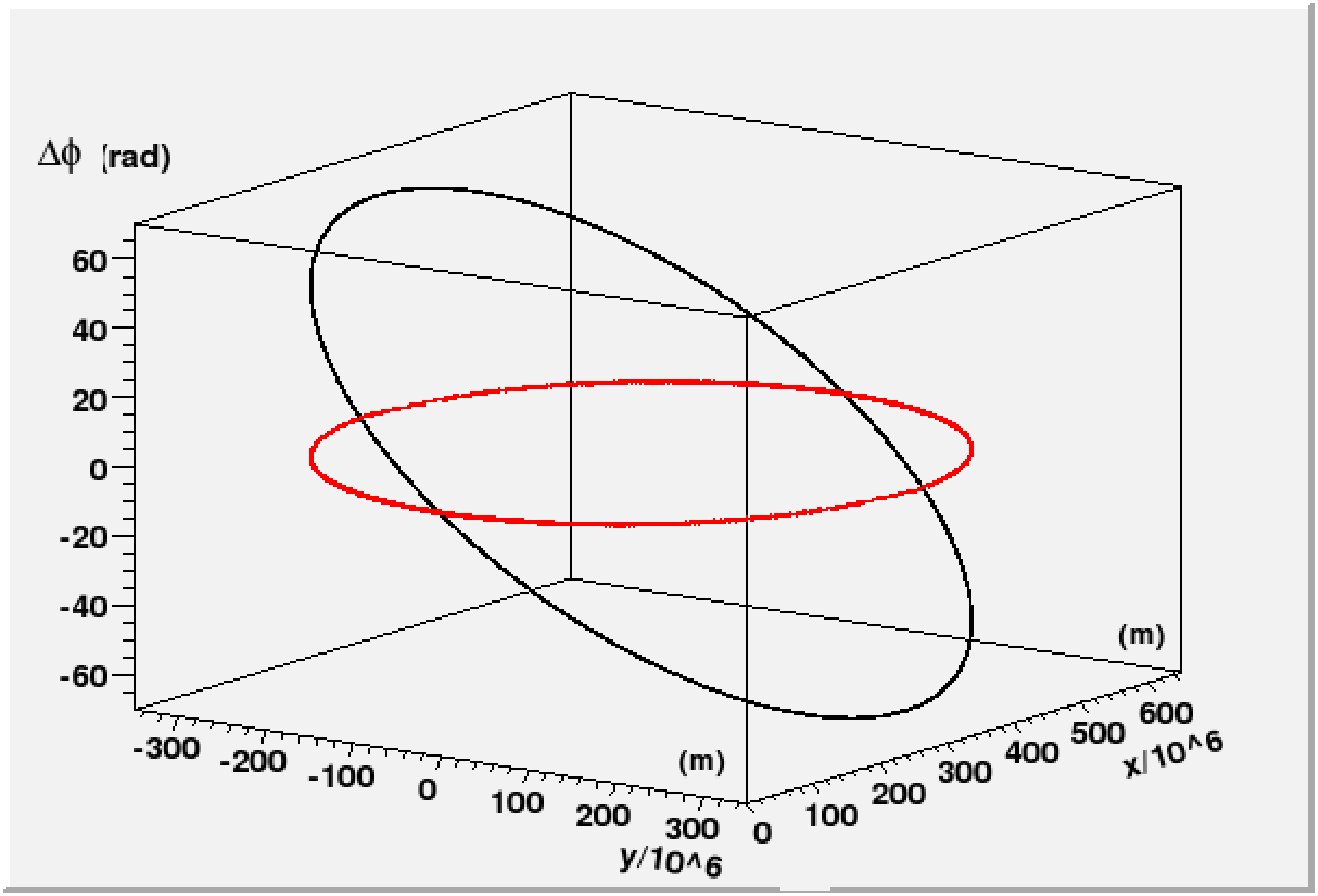
C) CORRECTOR

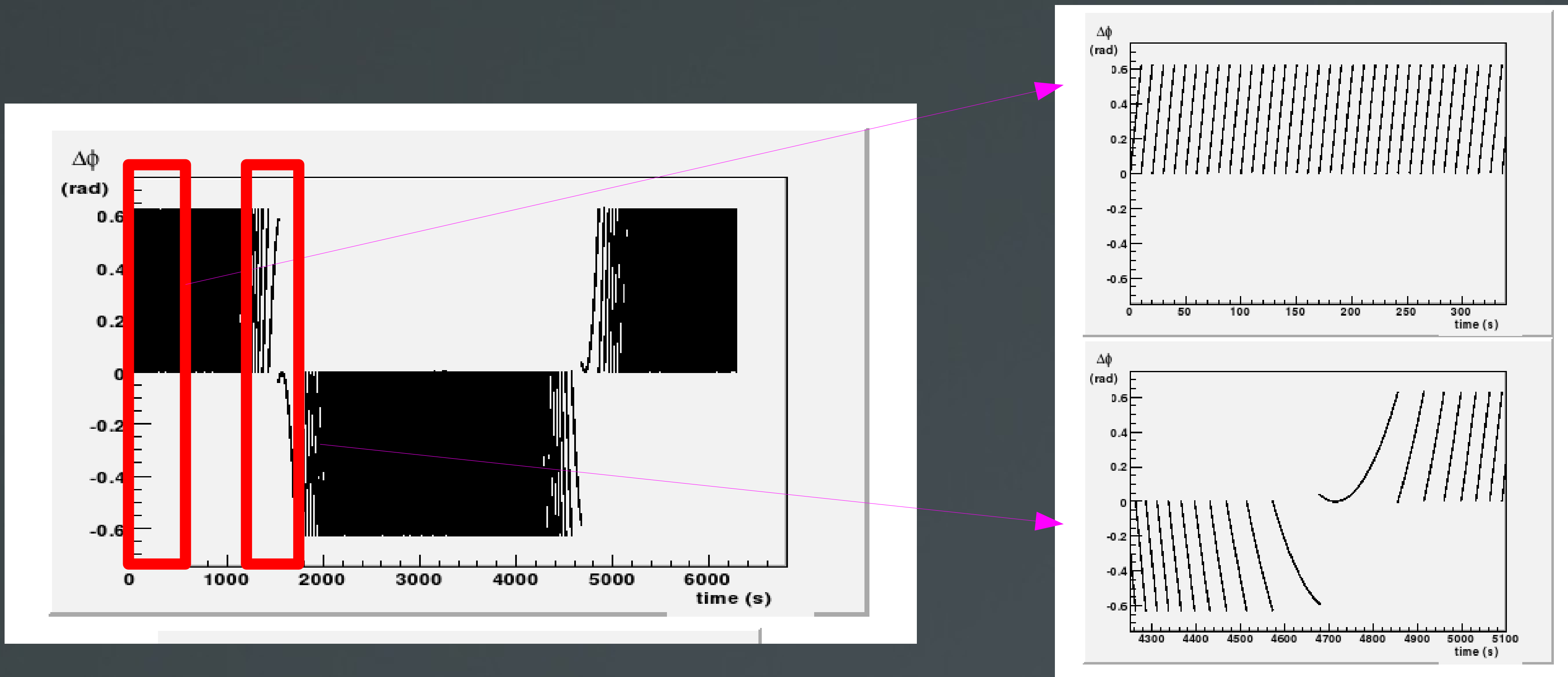
rest.dat
mov.dat
corr.dat
→ Analysis



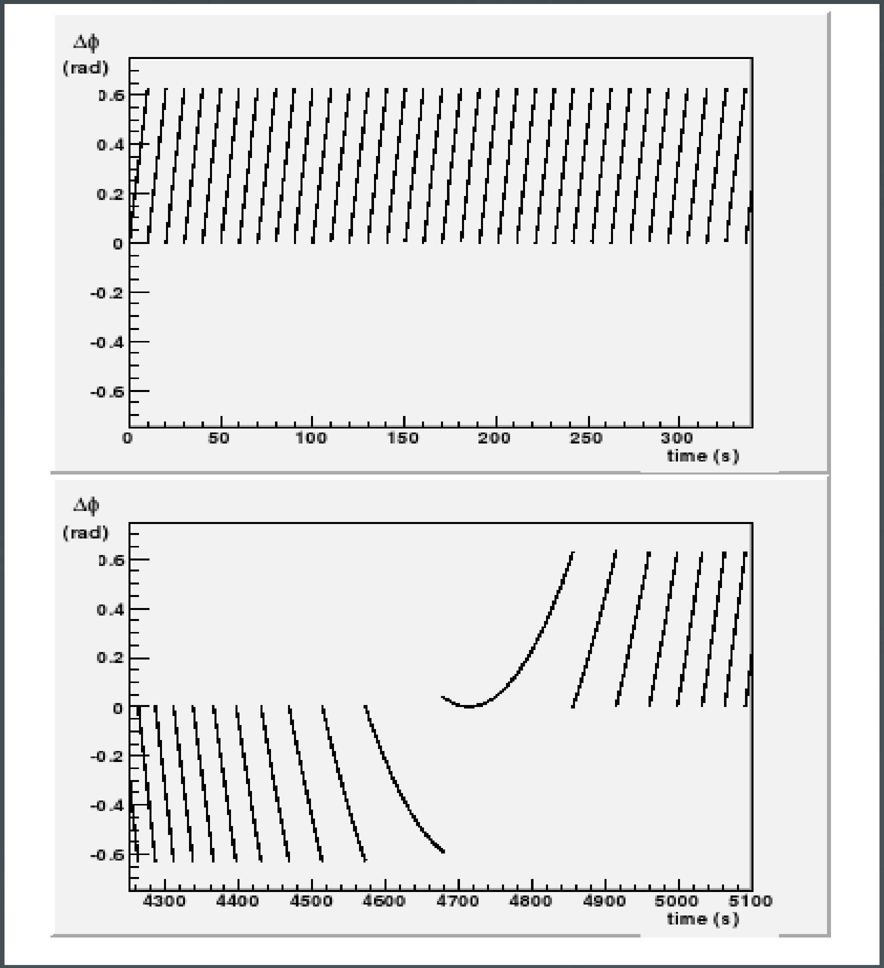
INPUT PARAMETERS: source frequency = 10 Hz, sampling frequency = 100 Hz, wave direction = (0,-1,0), orbital radius = 3 10^{8} m, β = 10^{-3} .

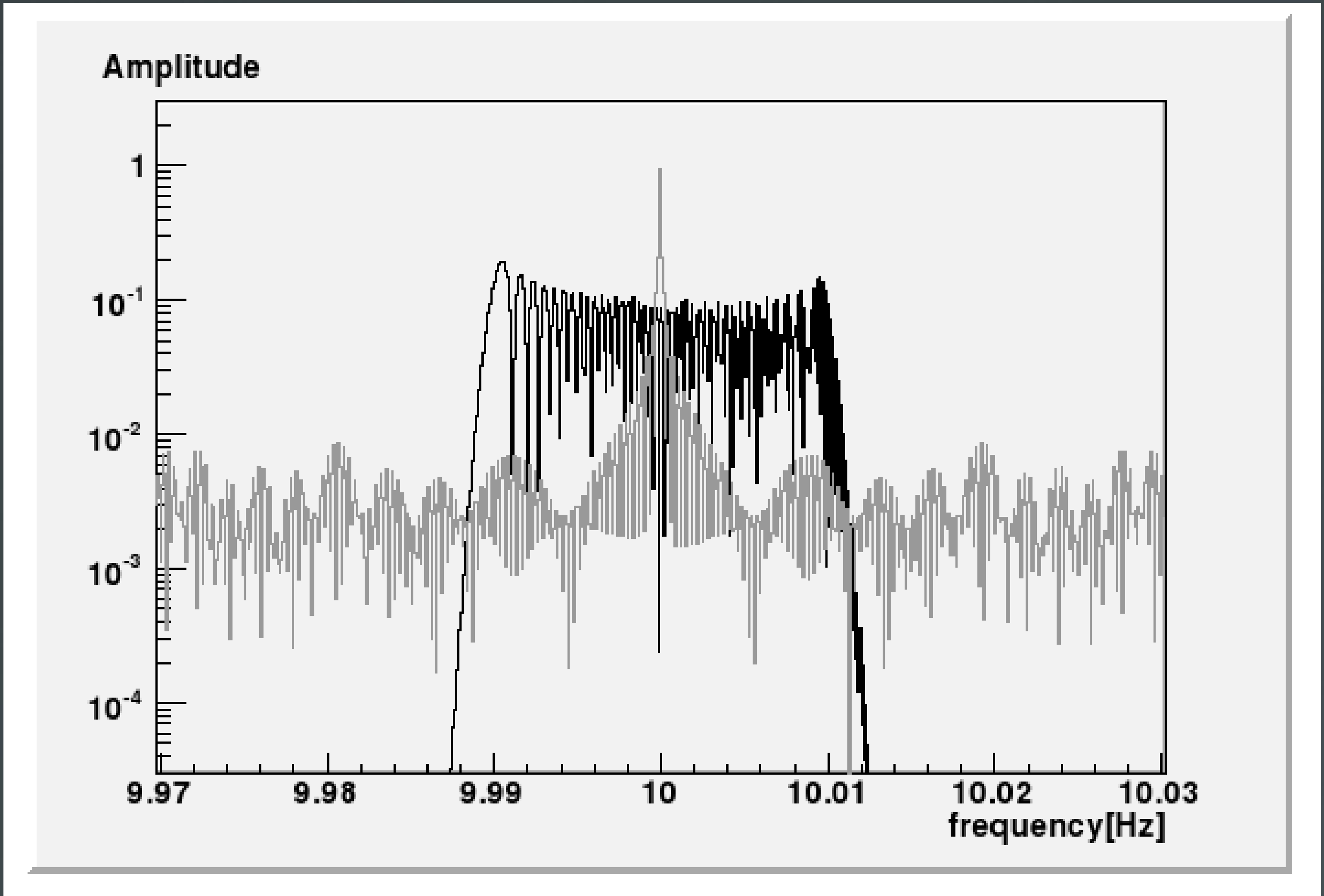


Phase difference between the sinuisodal wave as detected by the moviing observer with respect to rest observer as a function of the antenna orbital position x-y (black points). The same plot for the dephasing between the corrected signal and the one detected by the rest observer is displayed (red points).INPUT PARAMETERS: source frequency = 10 Hz, sampling frequency = 100 Hz, wave direction = (0,-1,0), orbital radius = 3 10^8 m, $\beta = 10^{-3}$.



Same phase difference in red points of previous fig, plotted as a function of time for the entire orbit (≃ 6, 283 s). Two zooms of the previous plot around t=0 and t=3/4 of the orbital period are reported.

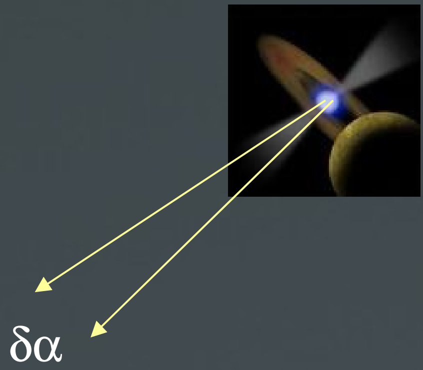

Δϕ
(rad)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
50
100
150
200
250
300
time (s)
Δϕ
(rad)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
4300
4400
4500
4600
4700
4800
4900
5000
5100
time (s)

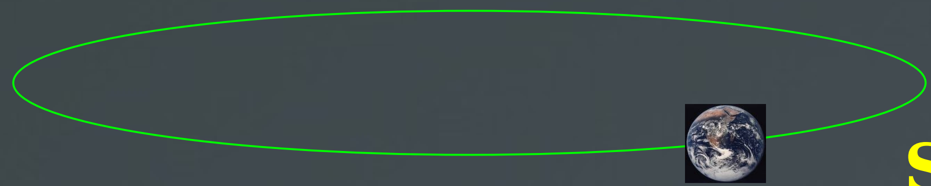


Linear spectral density of the signal as measured by moving observer (black) and of the corrected signal (gray)

$$\left(\vec{k}_1 - \vec{k}\right)\cdot\vec{r} = \frac{\omega_0 r}{c}\cos\beta\left[\left(\cos\delta\alpha - 1\right) + \sin\delta\alpha\sin\omega_y t\right]$$

Residual phase modulation due to direction error





Same parallel

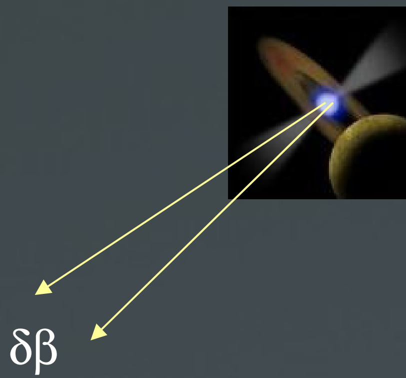
$$\frac{\omega_0 r}{c}\delta\alpha\cos\beta < 1 \Rightarrow \delta\alpha < \frac{c}{\omega_0 r\cos\beta}$$

Ask less than 1 rad (note f dependency)

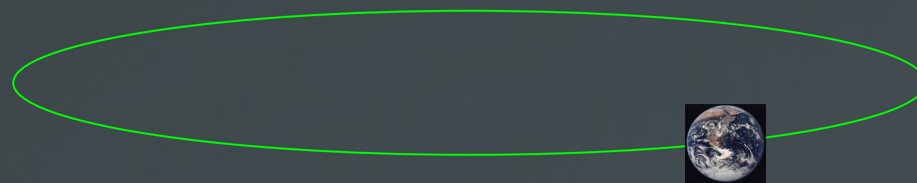
$$\delta\alpha < \frac{3\cdot 10^8}{1.5\cdot 10^{11}\cdot 2\pi 100} \simeq 3.18\cdot 10^{-6} rad = 0.66''$$

@ 100 Hz signal
Earth-Sun orbit

$$(\vec{k}_1 - \vec{k}) \cdot \vec{r} = \frac{\omega_0 r}{c} \cos \omega_y t \left[\cos \beta (\cos \delta\beta - 1) - \sin \beta \sin \delta\beta\right]$$



Phase modulation due to mistuning



Same meridian

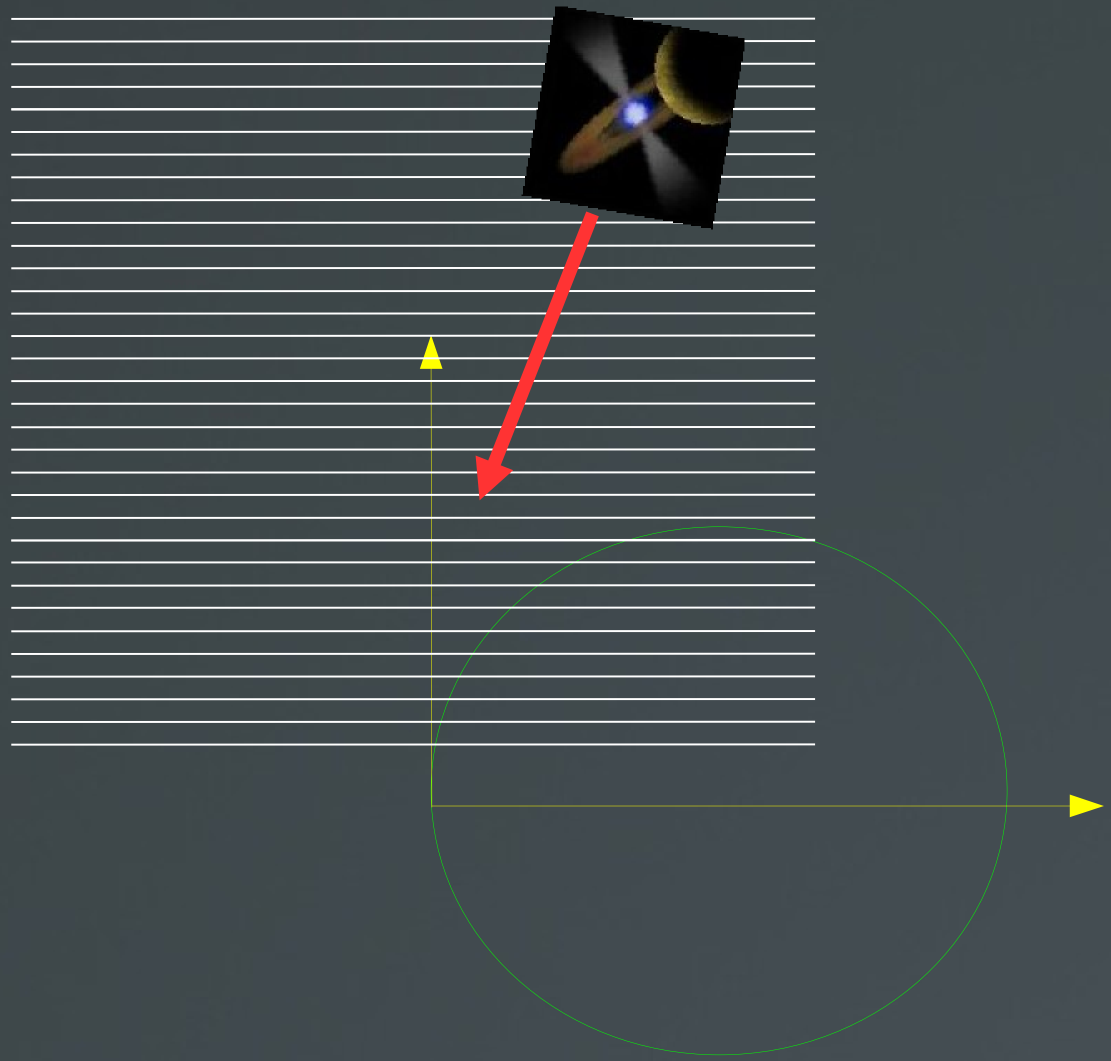
$$\frac{\omega_0 r}{c} \delta\beta |\sin \beta| < 1 \Rightarrow \delta\beta < \frac{c}{\omega_0 r |\sin \beta|}$$

Ask less than 1 rad (note f dependency)

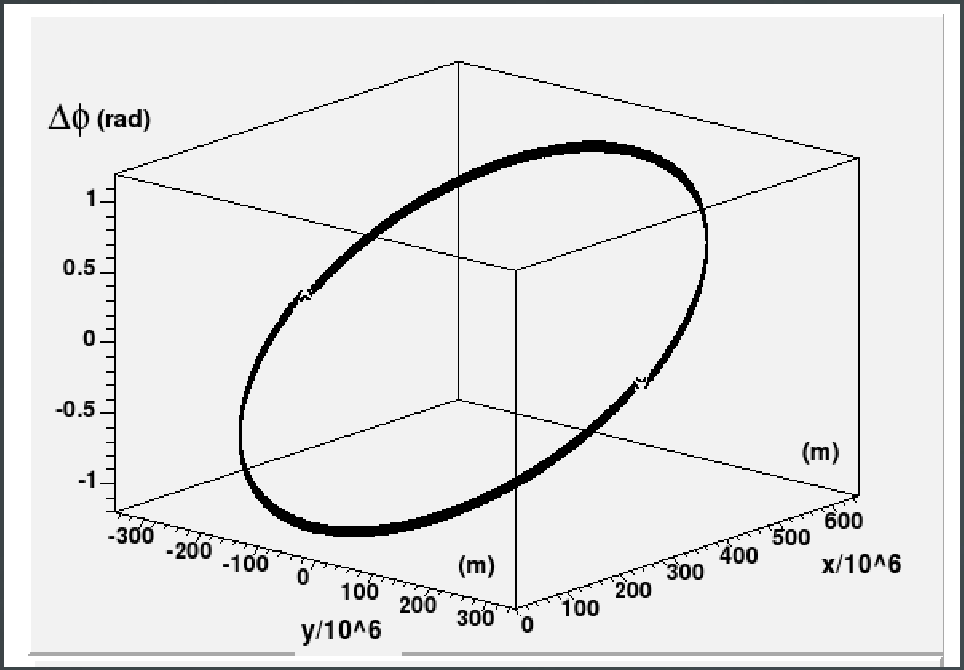
$$\delta\beta < 3.18 \cdot 10^{-6} rad = 0.66''$$

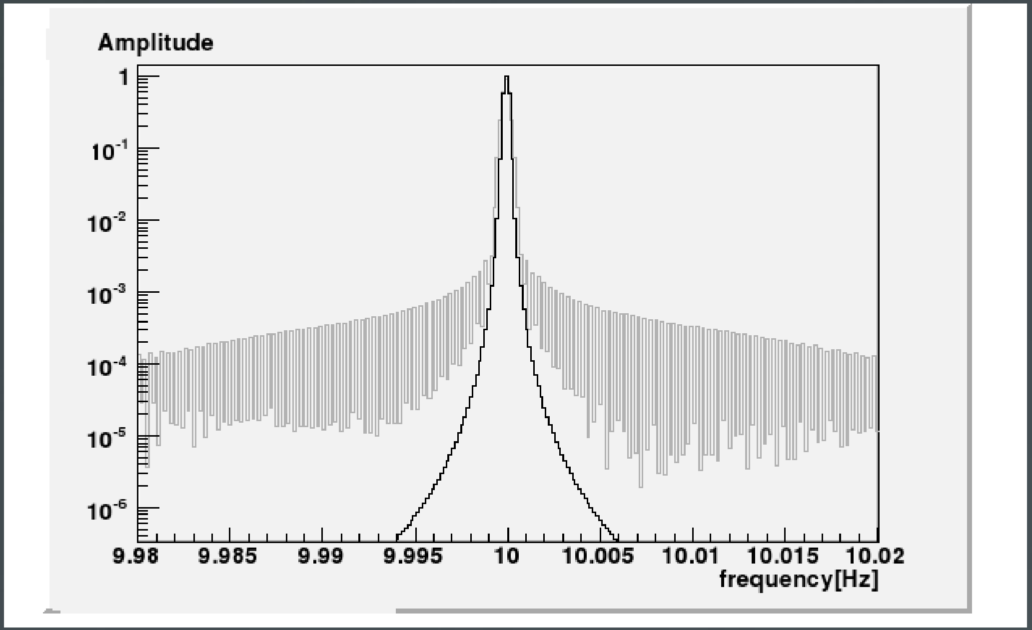
@ 100 Hz signal
Earth-Sun orbit

Symmetry in the two cases



Apply the discrete resampling method using a “wrong mask”

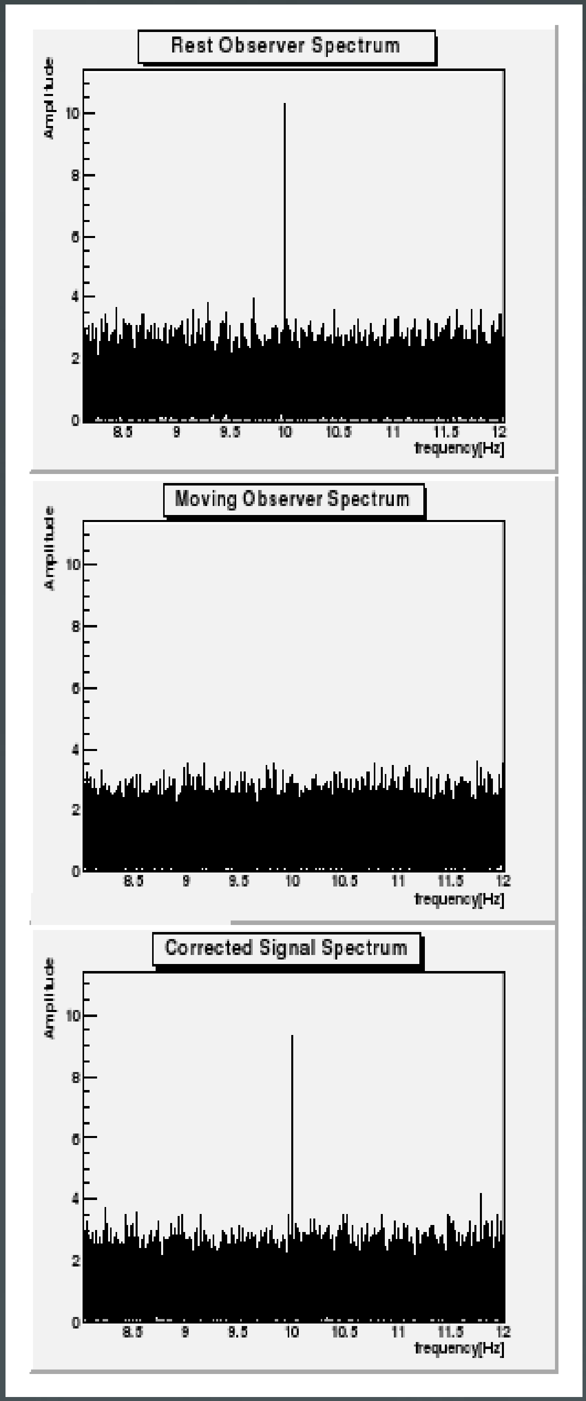




***Top**: Phase locking when the mask computed to correct for direction (0,-1,0) is applied to a signal coming from the same orbital plane, but inclined by 0.0157 rad anticlockwise with respect to the negative y direction.*

***Bottom**: Linear spectral density of the corrected signal (grey curve) and of the rest one (black) computed on one orbital period (about 6,283 s).*

Linear spectral density of the signal as measured by rest (top plot), moving (middle plot) and corrected (bottom plot) observer when a gaussian random noise is added to the monochromatic wave.



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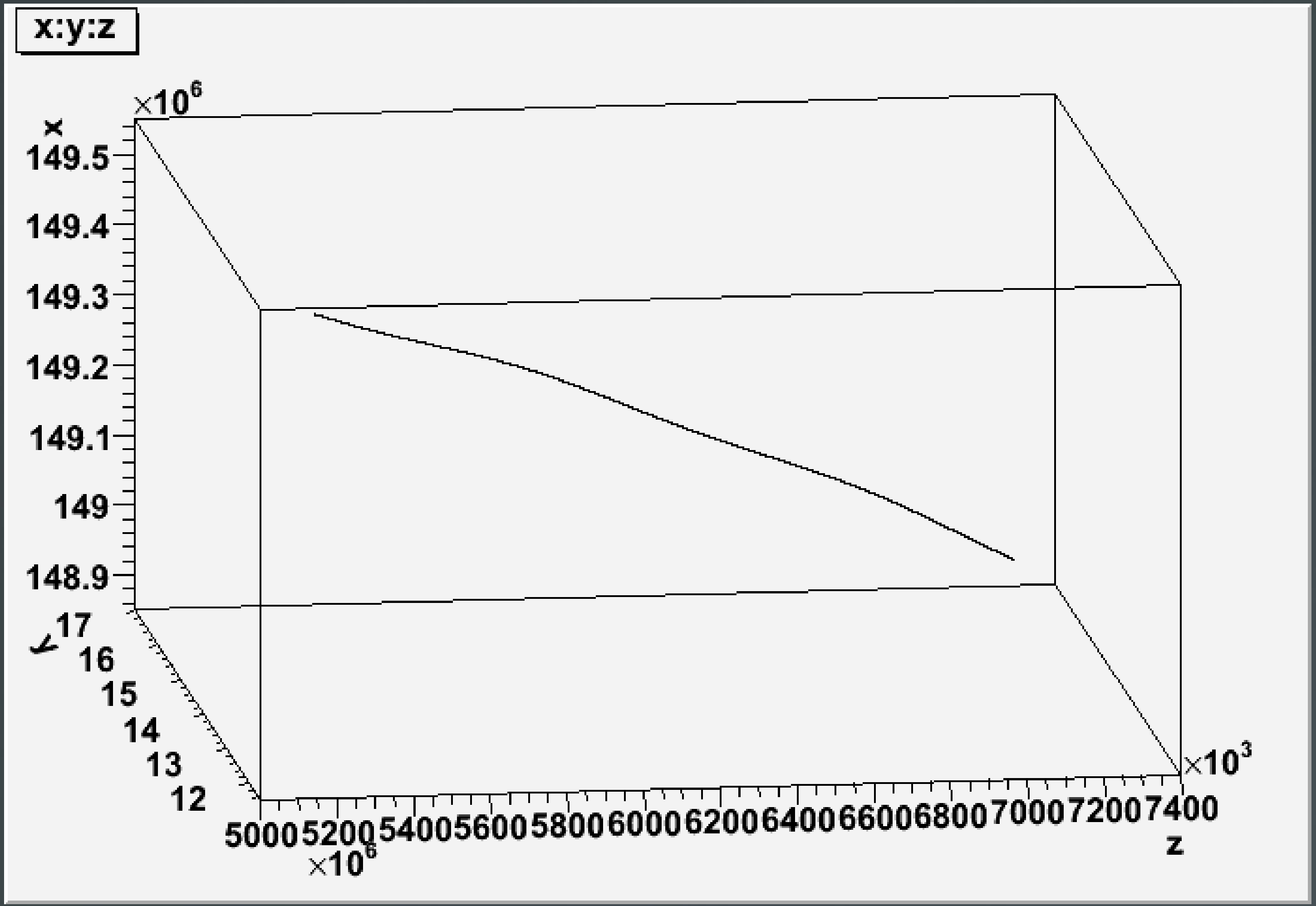
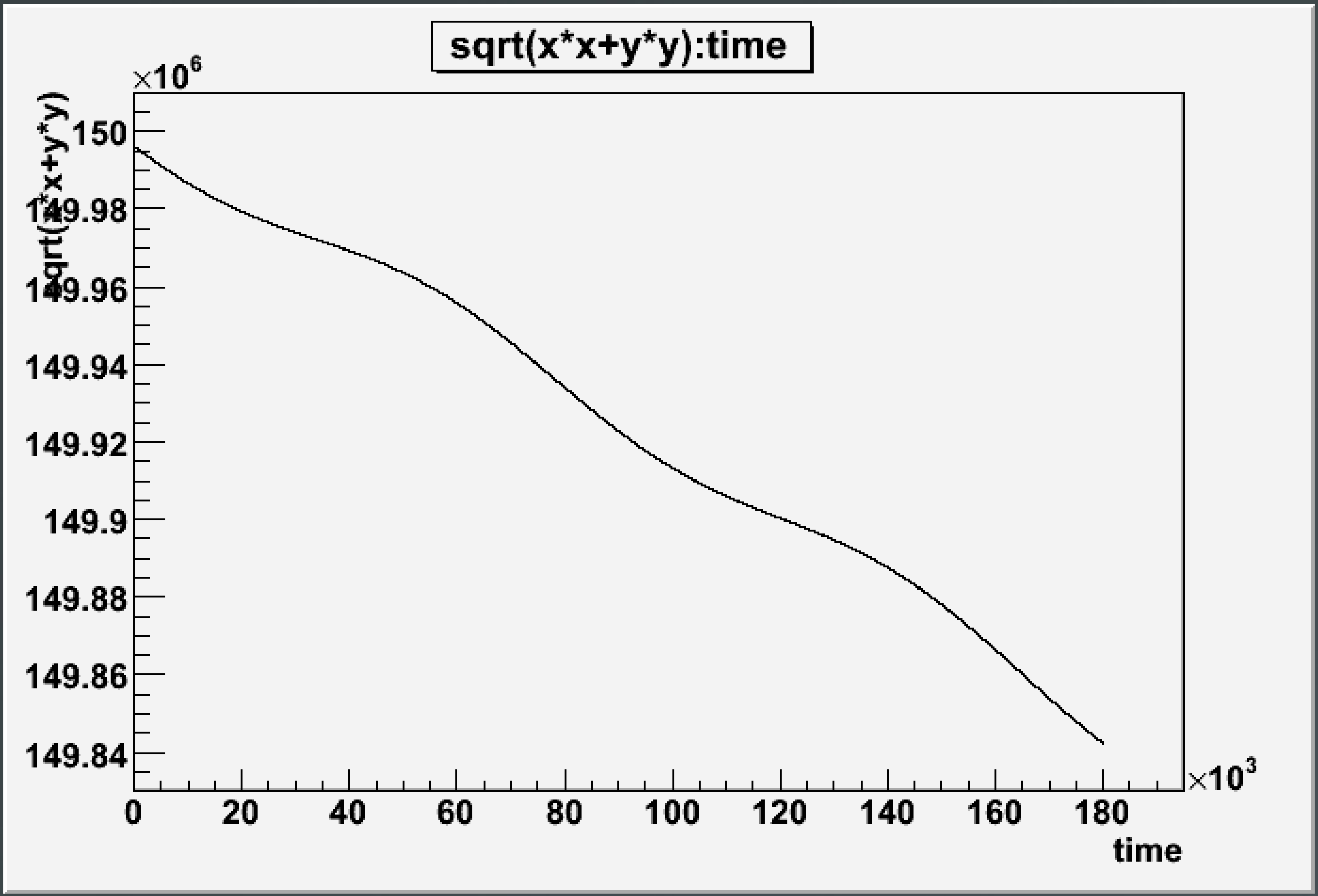


JPL
EPHEMERIDES

ANTENNA
COORDINATES

Roma 1 - PSS Software



JPL
EPHEMERIDES

ANTENNA
COORDINATES

11.6 s required computing time for 50.000 s at 2 Hz sampling frequency
(x,y,z,v_x,v_y,v_z) written on a file

Completely negligible........per thousands of real time

Next Step

Prepare the “mask” to put VIRGO VSR1 data “at rest”
with respect to the center of galaxy with different spin down factors.

LONG TERM FFT

HETERODYNE

...................TBD